A mathematical model for identifying an optimal waste management policy under uncertainty

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A B S T R A C T
In the municipal solid waste (MSW) management system, there are many uncertainties associated with the coefficients and their impact factors. Uncertainties can be normally presented as both membership functions and probabilistic distributions. This study develops a scenario-based fuzzy-stochastic quadratic programming (SFQP) model for identifying an optimal MSW management policy and for allowing dual uncertainties presented as probability distributions and fuzzy sets being communicated into the optimization process. It can also reflect the dynamics of uncertainties and decision processes under a complete set of scenarios. The developed method is applied to a case study of long-term MSW management and planning. The results indicate that reasonable solutions have been generated. They are useful for identifying desired waste-flow-allocation plans and making compromises among system cost, satisfaction degree, and constraint-violation risk.

1. Introduction
Rising municipal solid waste (MSW) generation rates, increasing environmental and health-impact concerns, shrinking waste-disposal capacities, and varying legislative and political conditions are providing significant challenges for waste managers. Moreover, in the MSW management problems, many system parameters and their interrelationships may appear uncertain. Many uncertainties can be presented in various stages of the policy cycle, ranging from the initial detection of a (possibilistic and/or probabilistic) problem, to policy formulation and, eventually, monitoring and adjustment to existing policies [1]. Moreover, these uncertainties may be further amplified by the complex features of the system components, as well as by their associations with economic penalties if the pre-regulated policies are violated. Therefore, in response to such complexities, effective MSW management methods are desired to be developed, by which sound management strategies with satisfactory economic and environmental efficiencies could be generated.

Previously, a large number of mathematical analysis methods were developed for supporting MSW management under uncertainty. Most of the studies focused on fuzzy mathematical programming (FMP) [2–4], interval mathematical programming (IMP) [5–6], stochastic mathematical programming (SMP) [7–11], and minimax regret (MMR) analysis methods [12,13]. For example, Jaung et al. [2] used fuzzy set theory to tackle decisions for siting landfills, where a procedure for systematic evaluation and ranking of prospective sites was provided. Huang et al. [14] proposed a violation analysis approach for the
planning of solid waste management systems under uncertainty, based on methods of interval-parameter fuzzy linear programming and regret analysis. Nie et al. [3] proposed an interval-parameter robust programming method for the planning of the solid waste management, where fuzzy robust linear programming and interval-parameter programming approaches were incorporated within an optimization framework to deal with interval numbers and fuzzy membership functions. Chang et al. [15] presented a fuzzy multicriteria decision analysis combined with a geospatial analysis for identifying landfill sites in a fast growing urban region. Generally, fuzzy linear programming (FLP) was useful when a model’s stipulations (i.e. right-hand-side values) were highly uncertain but with known membership functions. However, this approach was based on an assumption that the uncertain features of the model’s constraints were dependent upon each other, such that one control variable \((\lambda)\) was used for all constraints. This, however, might make some constraints not well satisfied while the others over-satisfied.

Fuzzy quadratic programming (FQP) could be effective for addressing the above deficiencies of FLP, which handled uncertainties for fuzzy constraints through using \(n\) control variables \((\lambda_i)\) corresponding to \(n\) respective constraints. For example, Tanaka and Guo [16] dealt with portfolio selection problems based on lower and upper possibility distributions and formulated them as quadratic programming models. Chen and Huang [17] developed an inexact fuzzy quadratic programming approach, through incorporating IMP and FQP within a general optimization framework to reflect uncertainties expressed as fuzzy sets and discrete intervals. Liu and Li [18] addressed a fuzzy quadratic assignment problem with penalty to minimize the total system cost through optimizing the job-allocation scheme. Li and Huang [10] advanced a fuzzy two-stage quadratic programming method to deal with dual uncertainties of fuzziness and randomness, which integrated FQP into two-stage stochastic programming (TSP) framework. However, this method could not adequately reflect the dynamic variations of system conditions (e.g., waste generation and allocation) for sequential structure of large-scale problems within a multistage context. For a MSW management system, the economic penalty subjected to uncertain waste-generation levels against pre-regulated policy targets is calculated using an optimization framework. """"But how"""". Nie et al. [3] proposed an interval-parameter robust programming method for the planning of the solid waste management, where fuzzy robust linear programming and interval-parameter programming approaches were incorporated within an optimization framework to deal with interval numbers and fuzzy membership functions. Chang et al. [15] presented a fuzzy multicriteria decision analysis combined with a geospatial analysis for identifying landfill sites in a fast growing urban region. Generally, fuzzy linear programming (FLP) was useful when a model’s stipulations (i.e. right-hand-side values) were highly uncertain but with known membership functions. However, this approach was based on an assumption that the uncertain features of the model’s constraints were dependent upon each other, such that one control variable \((\lambda)\) was used for all constraints. This, however, might make some constraints not well satisfied while the others over-satisfied.

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2. Methodology

Fuzzy linear programming (FLP) that is based on fuzzy set theory can deal with vague information in decision making problems. A FLP problem can be expressed as follows:

\[
\text{Min } f \equiv CX
\]

subject to:

\[
AX \leq B
\]

\[
X \geq 0
\]

where \(A = \{a_{ij}\}\) and \(A \in R^{m \times n} ; B = \{b_i\}\) and \(B \in R^{n \times 1} ; C = \{c_i\}\) and \(C \in R^{1 \times n} ; X = \{x_j\}\) and \(X \in R^{n \times 1} ; R\) denote a set of real numbers; symbols \(\equiv\) and \(\leq\) represent fuzzy equality and inequality. In fact, a decision in a fuzzy environment can be defined as the intersection of membership functions corresponding to fuzzy objective and constraints [22]. Given a fuzzy goal \((G)\) and a fuzzy constraint \((C)\) in a space of alternatives \((X)\), a fuzzy decision set \((D)\) can then be formed in the intersection of \(G\) and \(C\). In a symbolic form, \(D = G \cap C\), and correspondingly:

\[
\mu_D = \min\{\mu_G, \mu_C\}
\]

where \(\mu_D\), \(\mu_G\), and \(\mu_C\) denote membership functions of fuzzy decision \(D\), fuzzy goal \(G\), and fuzzy constraint \(C\), respectively. Let \(\mu_G(X)\) be membership functions of constraints \(C_i (i = 1, 2, \ldots, m)\), and \(\mu_G(X)\) be those of goals \(G_j (j = 1, 2, \ldots, n)\), a decision can then be defined by the following membership function [7]:

\[
\mu_D(X) = \mu_G(X) \cdot \mu_C(X)
\]

\[
\mu_D(X) = \min\{\mu_G(X)|i = 1, 2, \ldots, m + 1\}
\]

where \(X\) represent a set of fuzzy decision variables; """"\cdot"""" denotes an appropriate and possibly context-dependent """"aggregator""""; \(\mu_G(X)\) can be interpreted as the degree to which \(X\) satisfies fuzzy inequality in the objective and constraints. Thus, a
FLP problem can be converted into an ordinary linear programming one by introducing a new variable of \( \lambda = \mu_0(X) \), which corresponds to the membership function of the fuzzy decision [7,22].

Specifically, the flexibility in the constraints and fuzziness in the objective (which are represented by fuzzy sets and denoted as "fuzzy constraints" and "fuzzy goal", respectively) can be expressed as membership grade \( (\lambda) \) corresponding to the degree of overall satisfaction for the constraints and objective, which ranges between 0 and 1. A \( \lambda \) value close to 1 will correspond to a solution with a high possibility of satisfying the objective and constraints; conversely, a \( \lambda \) value near 0 will be related to a solution that has a low possibility of satisfying the objective and constraints. Thus, model (1) can be converted into:

\[
\begin{align*}
\text{Max } & \lambda \\
\text{subject to : } & \lambda P' + EX \leq B' + P' \\
& X \geq 0 \\
& 0 \leq \lambda \leq 1
\end{align*}
\]

in which:

\[
E = \begin{bmatrix} C \\ A \end{bmatrix}, \quad B' = \begin{bmatrix} f_1 \end{bmatrix} \quad \text{and } \quad P' = \begin{bmatrix} P_0 \\ P_1 \end{bmatrix}
\]

where \( P_0 \) and \( P_1 \) denote admissible violations of system objective and constraint \( i (i = 1, 2, \ldots, m) \); \( f_1 \) is the most desired objective function value. The essence of the above FLP formulation is that the "edges" of the feasible regions are not fixed. Each edge can be moved between two boundaries \( \sum a_{ij}x_j \leq b_i + P \) and \( \sum a_{ij}x_j \leq b_i + P_i \). The optimal solution is determined based on a compromise between having the objective approach the aspiration level \( (f_1) \) as closely as possible and having the minimum feasible region, formed by all \( \sum a_{ij}x_j \leq b_i \) \( (i = 1, 2, \ldots, m) \), be enlarged as slightly as possible [23]. The movement of all edges is controlled by a single variable \( (\lambda) \). Consequently, these edges of the feasible regions will move in the same direction and in an intercorrelated manner. This, in fact, implies an assumption that the fuzzy characteristics of the modeling constraints are dependent on each other. However, in many practical problems, they could be independent to each other (i.e. their boundaries can be moved from \( \sum a_{ij}x_j \leq b_i \) to \( \sum a_{ij}x_j \leq b_i + P_i \) independently). To address this problem, Cui and Blockley [23] suggested a fuzzy quadratic program (FQP), where \( m \) independent control variables \( \lambda_i, i = 1, 2, \ldots, m \) were associated with \( m \) fuzzy constraints, respectively. A linear membership function was adopted for the objective function, and parabolic membership functions were used for the constraints [23,24].

Let \( \lambda_0 \) denote the fuzziness in the objective function and \( \lambda_i, (i = 1, 2, \ldots, m) \) denote the fuzziness in constraint \( i \). The supports for the system objective and constraint \( i \) were then \( 1 - \lambda_0 \) and \( 1 - \lambda_i^2 \), respectively. An additive model proposed by Tiwari et al. [24] was adopted for generating optimal solutions by maximizing \( w_0 = 1 - \lambda_0 + \sum (1 - \lambda_i^2) \), where \( w_0 \) is a sum of all supports for the objective function and constraints. This is equivalent to minimizing \( w = \lambda_0 + \sum \lambda_i^2 \). Although the FQP’s criterion for determining the optimum is different from that of flexible FLP, its underlying meaning is the same as that in Zimmermann’s formula [22]. Thus, a FQP model for the above FLP problem can be formulated as follows:

\[
\begin{align*}
\text{Min } w &= \lambda_0 + \sum_{i=1}^{m} \lambda_i^2 \\
\text{subject to : } & \sum_{j=1}^{n} c_{ij}x_j + (1 - \lambda_0)(f_0 - f_1) \leq f_0 \\
& \sum_{j=1}^{n} a_{ij}x_j + \lambda_iP_i/2 \leq b_i + P_i/2, \quad i = 1, 2, \ldots, m \\
& x_j \geq 0, \quad j = 1, 2, \ldots, n \\
& 0 \leq \lambda_0 \leq 1 \\
& -1 \leq \lambda_i \leq 1, \quad i = 1, 2, \ldots, m
\end{align*}
\]

where \( f_0 \) and \( f_1 \) are the least and most desired objective function values, respectively, corresponding to control variable \( \lambda_0 \). The values of \( \lambda_1 \) to \( \lambda_m \) correspond to constraints 1 to \( m \) independently. When \( \lambda_i > 0 (i = 1, 2, \ldots, m) \), the boundary of constraint \( i \) can be moved inward closer to \( b_i \); when \( \lambda_i < 0 (i = 1, 2, \ldots, m) \), this boundary can be moved outward closer to \( b_i + P_i \). Thus, a lower \( \lambda_i^2 \) value represents a boundary closer to \( b_i + P_i/2 \), while a higher one corresponds to a boundary closer to either \( b_i \) or \( b_i + P_i \).

In FQP, multiple control variables (i.e. multiple \( \lambda \) levels) are employed to handle independent uncertainties in the model’s right-hand sides, and thus to enhance general satisfaction of the objective and constraints. However, it has difficulties in tackling uncertainties expressed as random variables in a non-fuzzy decision space and in providing a linkage between the pre-regulated policies and the associated implications. When uncertainties are expressed as random variables and the related study systems are of dynamic feature, the problem can be formulated as a scenario-based multistage stochastic programming (MSP) model as follows:
Min $f = \sum_{t=1}^{T} c_{1t} x_{1t} + \sum_{t=1}^{T} \sum_{k=1}^{K_{t}} p_{tk} d_{ik} y_{ik}$, \hspace{1cm} (6a)

subject to: \hspace{0.5cm} \begin{align*}
A_{0} x_{1t} &\leq b_{0t}, \quad r = 1, 2, \ldots, m_{1}; \quad t = 1, 2, \ldots, T, \hspace{1cm} (6b) \\
A_{0} x_{1t} + A'_{ik} y_{ik} &\leq w_{ik}, \quad i = 1, 2, \ldots, m_{2}; \quad t = 1, 2, \ldots, T; \quad k = 1, 2, \ldots, K_{t}, \hspace{1cm} (6c) \\
x_{pt} &\geq 0, \quad x_{pt} \in X_{pt}, \quad j = 1, 2, \ldots, n_{1}; \quad t = 1, 2, \ldots, T, \hspace{1cm} (6d) \\
y_{ikt} &\geq 0, \quad y_{ikt} \in Y_{ikt}, \quad j = 1, 2, \ldots, n_{2}; \quad t = 1, 2, \ldots, T; \quad k = 1, 2, \ldots, K_{t}, \hspace{1cm} (6e)
\end{align*}

where $p_{tk}$ is probability of occurrence for scenario $k$ in period $t$, with $p_{tk} > 0$ and $\sum_{k=1}^{K_{t}} p_{tk} = 1$; $D_{ik}$ are coefficients of recourse variables ($y_{ikt}$) in the objective function; $A_{ikt}$ are coefficients of $y_{ikt}$ in constraint $i$; $w_{ikt}$ is random variable of constraint $i$ with probability level $p_{tk}$; $K_{t}$ is number of scenarios in period $t$, with the total being $K = \sum_{t=1}^{T} K_{t}$. In the MSP model, decision variables are divided into two subsets: those that must be determined before the realizations of random variables are disclosed (i.e. $x_{pt}$), and those (recourse variables) that can be determined after the random-variable values are available (i.e. $y_{ikt}$).

Obviously, equations (6a)–(6e) can deal with uncertainties in the right-hand sides presented as random variables when coefficients in the left-hand sides and in the objective function are deterministic. Uncertainties in the above MSP model can be conceptualized into a multilayer scenario tree, with a one-to-one correspondence between the previous random variable and one of the nodes (states of the system) in each stage [21]. However, MSP is incapable of tackling uncertainties presented as fuzzy sets. Consequently, one potential approach for tackling uncertainties presented in terms of fuzzy sets, random variables, and their combinations is to incorporate the MSP and FQP within a general optimization framework. This leads to a scenario-based fuzzy-stochastic quadratic programming (SFQP) model as follows:

Min $w = \lambda_{0} + \sum_{t=1}^{T} \sum_{k=1}^{K_{t}} \left( \frac{1}{2} \right) p_{tk} \left( \beta_{ik}^{2} + \lambda_{f} (f_{i}^{u} - f_{i}^{l})^{2} \right) \hspace{1cm} (7a)$

subject to: \hspace{0.5cm} \begin{align*}
A_{0} x_{1t} &\leq b_{0t} + (1 - \lambda_{i}) \left( \frac{1}{2} \right) (B_{it}^{u} - B_{it}^{l})/2, \quad r = 1, 2, \ldots, m_{1}; \quad t = 1, 2, \ldots, T \hspace{1cm} (7b) \\
A_{0} x_{1t} + A_{ikt} y_{ikt} &\leq w_{ikt} + (1 - \lambda_{i}) \left( \frac{1}{2} \right) (\beta_{ikt}^{u} - \beta_{ikt}^{l}), \quad i = 1, 2, \ldots, m_{2}; \quad t = 1, 2, \ldots, T; \quad k = 1, 2, \ldots, K_{t} \hspace{1cm} (7c) \\
x_{pt} &\geq 0, \quad x_{pt} \in X_{pt}, \quad j = 1, 2, \ldots, n_{1}; \quad t = 1, 2, \ldots, T \hspace{1cm} (7d) \\
y_{ikt} &\geq 0, \quad y_{ikt} \in Y_{ikt}, \quad j = 1, 2, \ldots, n_{2}; \quad t = 1, 2, \ldots, T; \quad k = 1, 2, \ldots, K_{t} \hspace{1cm} (7e)
\end{align*}

In Eq. (7a), $\lambda_{0}$ is control variable corresponding to the membership grade of satisfaction for system objective; $\lambda_{i}$ and $\lambda_{i}$ are control variables corresponding to the membership grades of satisfaction for constraints of $r (r = 1, 2, \ldots, m_{1})$ and $i (i = 1, 2, \ldots, m_{2})$, respectively; the right-hand-side coefficients in equation (7c) are available as fuzzy sets, and the right-hand-side coefficients in equation (7d) are presented as fuzziness and randomness. Obviously, through introducing probability-density and fuzzy-membership functions into the system objective, the SFQP method can effectively tackle dual uncertainties presented as fuzziness and randomness; furthermore, it can reflect dynamic feature of the system conditions through transactions at discrete points in time over a multistage context.

3. Case study

3.1. Statement of problems

A MSW management problem is developed for demonstrating the proposed SFQP method. Consider a manager is responsible for allocating MSW flows from one city to three waste-management facilities over a long-term planning horizon, as shown in Fig. 1. The MSW management system involves in a series of processes such as waste generation, storage, collection, transportation, treatment and disposal. Moreover, various factors should also be taken into account by decision makers including the collection techniques to be used, the level of service to be offered, and the facilities to be adopted. The processes and factors are complex with multi-period, multi-facility, multi-layer, multi-uncertainty and multi-objective features. The study system is composed of one landfill, one recycling program, and one composting facility. Currently, the city’s majority of MSW flows disposed of at the landfill, and the amount of waste flows allocated to the recycling and composting facilities (i.e. diverted from the landfill) is relatively low. However, the landfill can release a wide range of chemicals resulting from the waste degradation in the forms of leachate, gas and degraded waste [25–27]; this may pose serious risks on the surrounding environment and the local public health. The scarcity of land near urban centers and the growing opposition from the public force the local authority to make efforts to develop a policy guidance for MSW diversion and recycling, which can be used to reduce the amount of waste that ends up at the landfill. Therefore, establishment of regulated waste diversion targets and relevant regulations is desired.

In the study system, uncertainties can be normally presented as both membership functions and probabilistic distributions (e.g., vagueness existing in the outcomes of a random event). For example, waste generation rate may be highly
uncertain in nature since it is affected directly by a number of factors, such as economic development, population growth, human activities, and public habits. Uncertainties in waste-generation rate may contain not only randomness with probability distributions but also fuzziness in individual events (of the realized waste) with varied probability levels. For instance, when being asked for the waste-generation amount in one city, the decision maker may respond that “the daily waste generation of the residents is probably 250–275 tonnes”; this results in dual uncertainties of vagueness and randomness in the waste-generation rate. Table 1 presents the waste-generation rates and the associated probabilities of occurrence in the planning periods. The planning horizon is 15 years with three 5-year periods. The waste-generation rates are presented as random variables; meanwhile, some random events may also contain vague information, leading to dual uncertainties (e.g., waste generation rate is probably 1373–1433 tonnes per week associated with probability of 12.5% in period 1). Moreover, the capacity of each waste management facility could fluctuate with fuzzy feature due to the existence of many uncertainties and complexities, such as (a) variations in working hours, (b) requirements for system maintenance, and (c) inconsistent manners among workers in operating the facility. In this system, the landfill has a capacity of possibly from 0.72 to 0.80 million tonnes, the composting facility has a capacity in the range of 480–560 tonne/week, and the recycling facility has a capacity to process 420–470 tonnes waste per week. Besides, the composting facility generates residues of approximately 12% (on a mass basis) of incoming waste streams, and the recycling facility generates approximately 8% residues. All of the residues are disposed of at the landfill.

Since the waste-generation rates are highly uncertain (i.e. containing fuzzy and random information), a projected waste-flow level needs to be pre-regulated based on the city’s waste management policy. Correspondingly, an allowable waste-flow level from the city to each facility is pre-regulated. If this level is not exceeded, it will result in a normal (regular) cost to the system. However, if it is exceeded, the surplus flow should be disposed of expensively, resulting in an excess cost (penalty) to the system. Under such a situation, the total waste-flow amount will be the sum of both fixed allowable and probabilistic surplus flows. The total cost includes the regular costs (for disposing of allowable waste flows) and possibilistic penalties (for treating excess waste flows). Since the relationship between waste-generation rates and available facility capacities are varying, the optimal schemes for effective utilization of the facilities (i.e. optimal waste-flow allocation patterns) will also change in different time periods. Table 2 shows the relevant waste diversion goals (as required by the authorities) as well as the minimum and maximum allowable waste flows. Based on the city’s waste management policy, 50% of waste diverted from the landfill will be achievable within the planning horizon. The allowable waste flows to the landfill will be regulated with dynamic reduction in correspondence with the increasing waste diversion goal along with time.

Table 3 provides collection and transportation costs for allowable and excess waste flows to the landfilling, composting and recycling facilities, operating costs of the three facilities, penalty costs for surplus waste flows, and revenues from the composting and recycling facilities. Costs for waste collection and transportation are estimated based on the existing conditions in the collection areas; the average container size, collection frequency, collection mode (automatic and manual), and collection time (per load) were also considered when making the estimates. Moreover, penalty is much higher than the normal cost. The surplus waste flow (i.e. when the pre-regulated allowable waste level is exceeded) will be disposed of at a premium, resulting in a raised cost (penalty) to the system. The raised cost is mainly due to more expensive labor
and facility operation for the collecting, transporting and treating excess waste. In detail, such raised costs (economic penalties) are associated with: (i) increased collection cost for excess waste (e.g., longer time and more workers are required for collecting the surplus waste), (ii) increased transportation cost for shipping the excess waste to more remote facilities (when the capacities of local facilities are exhausted), (iii) increased operating costs for waste management facilities (e.g., extended working hours, more workers, and more expensive facilities), and (iv) extra expenses and/or fines caused by contingent events.

### 3.2. Modeling formulation

A number of complexities exist in the study system such as uncertainties in waste-management facilities, dynamic variation in system components, randomness and fuzziness in waste-generation rates, policy analysis for waste-flow allocation, objectives in economic and environment, as well as requirements for waste diversion. The waste manager desires to achieve a minimum expected value of total cost for disposing of MSW in the region. The study problem can be formulated as a SFQP model as follows:

\[ \text{Min } w = \lambda_0 + \sum_{i=1}^{3} \sum_{k=1}^{K_i} L_{ik} (TR_{it} + OP_{it}) + \sum_{i=1}^{3} \sum_{k=1}^{K_i} p_{ik} (\lambda_{it}^W)^2 + \sum_{i=1}^{3} \sum_{k=1}^{K_i} p_{ik} (\lambda_{it}^D)^2 \]  \hspace{1cm} (8a)

subject to:

\[ \sum_{i=1}^{3} L_{it}X_{it} [TR_{it} + OP_{it} + FE_i (FT_{it} + OP_{it}) - RE_{it}] \]
\[ + \sum_{i=1}^{3} \sum_{k=1}^{K_i} L_{ik} p_{ik} Y_{itk} (DR_{it} + DP_{it}) \]
\[ + \sum_{i=2}^{3} \sum_{k=1}^{K_i} L_{ik} p_{ik} Y_{itk} (DR_{it} + DP_{it} + FE_i (DT_{it} + DP_{it}) - RM_{it}) \leq f^d + f_0 (f^d - f^l) \]  \hspace{1cm} (8b)

[System objective constraint]

\[ \sum_{i=1}^{3} L_{it} \left( X_{it} + Y_{itk} \right) + \sum_{i=2}^{3} \sum_{k=1}^{K_i} FE_i (X_{it} + Y_{itk}) \leq LC^i + \frac{(1 - \lambda_{itk})(LC^j - LC^l)/2}{k = 1, 2, \ldots, K_i} \]  \hspace{1cm} (8c)
\[ X_{it} + Y_{itk} \leq TC_i^L + (1 - \lambda_{itk})(TC_i^U - TC_i^L)/2, \quad \forall t; i = 2, 3; k = 1, 2, \ldots, K_i \]  

[Constraints of waste-management-facility capacity]

\[ \sum_{i=1}^{3}(X_{it} + Y_{itk}) = \overline{WG}_{ik} + (1 - \lambda_{itk})(\overline{W}\overline{G}_{ik}^U - \overline{W}\overline{G}_{ik}^L)/2, \quad \forall t; k = 1, 2, \ldots, K_i \]  

[Constraint of waste disposal demand]

\[ X_{it} + Y_{itk} \leq DC_{it} \overline{WG}_{ik} + (1 - \lambda_{itk}) (DC_{it} \overline{WG}_{ik}^U - DC_{it} \overline{WG}_{ik}^L), \quad \forall t; k = 1, 2, \ldots, K_i \]  

[Constraint for waste-diversion requirement]

\[ A_{itk}^L \leq X_{itk} \leq A_{itk}^U, \quad \forall i, t \]  

[Constraint for allowable waste flow]

\[ 0 \leq Y_{itk} \leq X_{itk}, \quad \forall i, t; k = 1, 2, \ldots, K_i \]  

[Constraint for excess waste flow]

where \( i \) is type of waste management facility, with \( i = 1 \) for landfill, \( i = 2 \) for composting facility, and \( i = 3 \) for recycling facility; \( t \) is time period, and \( T = 3 \); \( L \) denotes length of time period \( t \) (week); \( f^L \) and \( f^U \) are lower and upper bounds of the desired system cost; \( OP_{it} \) and \( DP_{it} \) are operating costs of facility \( i \) for allowable and excess waste flows during period \( t \) ($/t), where \( DP_{it} \geq OP_{it} \); \( TR_{it} \) and \( DR_{it} \) are collection and transportation costs for allowable and excess waste flows to facility \( i \) during period \( t \) ($/tonne), where \( DR_{it} \geq TR_{it} \); \( FT_{it} \) and \( DT_{it} \) are transportation costs for allowable and excess waste residues from facility \( i \) to the landfill during period \( t \) ($/tonne), where \( DT_{it} \geq FT_{it} \) and \( i = 2, 3; FE \) denotes residue flow rate from facility \( i \) to the landfill (% of incoming mass to facility \( i \)), \( i = 2, 3; LC^L \) and \( LC^U \) are lower- and upper-bound landfill capacity (tonne); \( K_t \) is the number of waste-generation scenarios in period \( t \), with the total number of scenarios being \( K = \sum_{t=1}^{T} K_t \); \( p_{th} \) is the probability of occurrence for waste generation in period \( t \) under scenario \( k \), with \( p_{th} > 0 \) and \( \sum_{k=1}^{K} p_{th} = 1 \); \( RE_{it} \) and \( RM_{it} \) are revenues from composting and recycling facilities because of allowable and excess flows during period \( t \) ($/tonne); \( TC_i^L \) and \( TC_i^U \) are lower- and upper-bound capacities of composting and recycling facilities (tonne/week); \( A_{itk}^L \) and \( A_{itk}^U \) are the lower and upper allowable waste flows to facility \( i \) during period \( t \) (tonne/week); \( W\overline{G}_{ik} \) and \( W\overline{G}_{ik}^U \) are random waste generation rates of lower and upper bounds in period \( t \) under scenario \( k \) (tonne/week), and their associated probabilities of occurrence are \( p_{th} \), \( w \) denotes a general satisfaction degree for both system objective and constraints; \( \lambda_{itk} \) is the control variable corresponding to the system objective constraint; \( \lambda_{itk} \) are control variables related to the constraints of waste-management-facility capacity; \( \lambda_{itk}^W \) are control variables related to the constraints of waste generation levels; \( \lambda_{itk}^U \) are control variables associated with the constraints of waste-diversion requirement; \( X_{it} \) is allowable waste flow as pre-regulated by the authority to facility \( i \) during period \( t \) (tonne/week) (the first-stage decision variable); \( Y_{itk} \) is the amount by which the allowable waste level \( (X_{it}) \) is exceeded when the waste-generation rate is \( [\overline{W}\overline{G}_{ik}, \overline{W}\overline{G}_{ik}^U] \) with probability \( p_{th} \) under scenario \( k \) (tonne/week) (the recourse decision variable).

4. Results and discussion

4.1. Results analysis

In the above SFQP model, a multilayer scenario tree was constructed for reflecting uncertain waste-generation levels, resulting in a total of 52 scenarios (i.e., 4 scenarios in period 1, 12 scenarios in period 2, and 36 scenarios in period 3). Scenario 1 denotes low waste-generation rate in period 1 with a probability of 12.5%; scenario 52 corresponds to high waste-generation rates in the three periods with a joint probability of 0.93%. Interpretation and analysis for the solutions obtained from the models are provided below. Fig. 2 provides the solutions for waste-flow-allocation patterns from the SFQP model; they include allowable and excess flows from the city to the landfill, composting and recycling facilities over the planning horizon. For example, in period 1, the optimized allowable waste flows to the landfill would be 950.0 tonne/week; when waste-generation rates are low, low-medium, medium and high, there would be excess flows of 63.9, 124.9, 196.7 and 208.7 tonne/week (in reference to the allowable waste-loading level) allocated to the landfill, respectively; correspondingly, the total flows to the landfill would increase to 1013.9, 1074.9, 1146.7 and 1158.7 t/wk under the four waste-generation levels, respectively. The solutions for the other facilities and scenarios can be similarly interpreted based on the results as shown in Fig. 2, respectively. Generally, the results from the SFQP indicate that (i) an excess flow could be generated if the allowable waste level was exceeded (i.e. excess flow = generated flow – allowable flow); (ii) the waste flow-allocation patterns would vary under different scenarios, due to the temporal and spatial variations of waste generation and management conditions; (iii) the waste flows to the landfill would be decreasing along with time while those to the composting and recycling facilities would keep increasing, to satisfy the required diversion goal; (iv) in the case of excess waste, the allotment to the landfill would be assigned initially within the diversion goal (i.e. satisfying 50% of diversion requirement at the end of planning horizon).
Fig. 3 provides the results for waste-flow-allocation patterns from SFQP and SFQP-N models. The SFQP-N model merely focused on minimizing the total cost for waste management, while constraint for waste diversion (i.e. Eq. (8f)) was not required in the SFQP-N. In comparison, in SFQP, an increasing waste-diversion rate and thus reducing waste flows to the landfill with a minimized total cost were concerned. The results indicate that much more waste flows would be allotted to the landfill from the SFQP-N. This is because the composting and recycling facilities have higher regular and penalty costs for treating wastes, allotment of wastes to the landfill would be more economical without consideration of waste-diversion requirement. For instance, when waste-generation rates are low over the planning horizon, the waste flows shipped to the landfill would be 4871.1 \times 10^3 tonnes (from SFQP) and 5046.9 \times 10^3 tonnes (from SFQP-N); when waste-generation rates are high in all of the three periods, the waste flows to the landfill would be 5189.5 \times 10^3 tonnes (from SFQP) and 5364.9 \times 10^3 tonnes (from SFQP-N).

Fig. 4(a) shows the effects of landfill-capacity variation on the total cost and the corresponding satisfaction degree under SFQP. Several situations under varied landfill-capacity constraints are analyzed. As the landfill capacity decreases, the total cost would increase but, the corresponding satisfaction degree would decrease. For example, when the admissible interval of landfill capacity is [0.82,0.90] million tonnes (i.e. the lower-bound landfill capacity is 0.82 million tonnes as shown in Fig. 4(a)), the results from SFQP indicate that the total cost would be $69.8 \times 10^6 with satisfaction degree of 0.55; when the landfill capacity is [0.68,0.76] (i.e. the lower landfill capacity is 0.68 million tonnes), the total cost would be $73.6 \times 10^6 and the satisfaction degree would be zero. Fig. 4(b) shows the total costs and satisfaction degrees from SFQP-N. When the landfill capacity is [0.82,0.90] million tonnes, the total cost from SFQP-N would be $69.0 \times 10^6 with satisfaction degree of 0.63; when the landfill capacity is [0.78,0.86], the total cost from SFQP-N would be $69.3 \times 10^6 with satisfaction degree of 0.55. As the landfill capacity decreases, the total cost from SFQP-N would raise and the corresponding satisfaction degree would reduce. In addition, the results also indicate that, under the majority of conditions, SFQP-N has a lower cost and a higher satisfaction degree, compared with SFQP; however, when the landfill capacity is low, SFQP and SFQP-N would have the same cost and satisfaction degree (i.e. the constraint for waste diversion is insignificant under such a condition). The cost for waste diversion (to composting and recycling facilities) is much higher than that for waste disposal of at the landfill, leading to a high cost under SFQP-N. However, without consideration of waste diversion, more land-resource consumption and pollutant emission from the landfill could bring about higher negative consequences than the savings obtained from SFQP-N.
Fig. 3. Comparison of waste-flow-allocation patterns between SFQP and SFQP-N.

Fig. 4. Sensitivity analysis for the effect of landfill-capacity variations.
4.2. Comparison with best/worst case analyses

Simplifying the fuzzy sets (in the above SFQP model) into two extreme values (i.e., lower and upper bounds), the study problem can be solved through two best/worst case models (abbreviated as BCM and WCM), based on the scenario-based multistage stochastic linear programming technique. BCM corresponds to the upper waste-management facilities and the lower waste-generation rates, while WCM is associated with the lower waste-management capacities and the upper waste-generation rates. Thus, we have:

Min \( f = \sum_{t=1}^{T} L_t X_{1t}(TR_{1t} + OP_{1t}) + \sum_{i=2}^{3} \sum_{t=1}^{T} L_t X_{it}[TR_{it} + OP_{it} + FE_i(FT_{it} + OP_{it})] - RE_{it} \)

+ \( \sum_{i=1}^{K} \sum_{k=1}^{K} L_t X_{1tk}(DR_{1tk} + DP_{1tk}) + \sum_{i=2}^{3} \sum_{k=1}^{K} L_t X_{itk}[DR_{itk} + DP_{itk} + FE_i(DT_{itk} + DP_{itk})] - RM_{itk} \),

subject to:

\( \sum_{t=1}^{T} L_t \left[ (X_{it} + Y_{itk}) + \sum_{i=2}^{3} FE_i(X_{it} + Y_{itk}) \right] \leq LC(L,U), \quad \forall t; \quad k = 1, 2, \ldots, K \),

\( X_{it} + Y_{itk} \leq TC_i(L,U), \quad \forall t; \quad i = 2, 3; \quad k = 1, 2, \ldots, K \),

\( \sum_{i=1}^{3} (X_{it} + Y_{itk}) = W G_k(L,U), \quad \forall t; \quad k = 1, 2, \ldots, K \),

\( X_{it} + Y_{itk} \leq DG_{it} W G_k(L,U), \quad \forall t; \quad k = 1, 2, \ldots, K \),

\( A_{it}^0 \leq X_{it} \leq A_{it}^U, \quad \forall t \),

\( 0 \leq Y_{itk} \leq X_{it}, \quad \forall i, t; \quad k = 1, 2, \ldots, K \).

Fig. 5 provides the solutions for the optimized allowable and excess flows obtained through solving the BCM and WCM models; they are significantly different from each other. For example, in period 1, the solutions of BCM indicate that the

![Graphs showing waste flow allocation patterns](image-url)
optimized allowable flows to the landfill, composting and recycling facilities would be 950.0, 200.0 and 200.0 tonne/week, respectively; in comparison, the solutions of WCM indicate that the optimized allowable flows to the three facilities would be 950.0, 200.0 and 283.0 tonne/week, respectively. For excess flows, when waste generation rate is high in period 1, the solutions of BCM indicate that the excess flows to the landfill, composting and recycling facilities would be 228.0, 0 and 0 tonne/week, respectively, while excess flows (obtained from WCM) to the three facilities would be 122.3, 92.7 and 0 tonne/week, respectively. Consequently, the total flows (from BCM) to the landfill, composting and recycling facilities would be 1178.0, 200.0 and 200.0 tonne/week, respectively, while those (from WCM) to the three facilities would be 1072.3, 292.7 and 283.0 tonne/week, respectively.

Fig. 6 presents a comparison of waste-flow-allocation patterns from SFQP, BCM and WCM. The resulting waste-flow allocation patterns are different from each other. For example, when waste-generation rates are low in all of the three periods (denoted as symbol LLL), the waste flows disposed of by the landfill would be $4871.1 \times 10^3$ tonnes under SFQP, $4855.3 \times 10^3$ tonnes under BCM, and $4655.6 \times 10^3$ tonnes under WCM; those treated by the composting facility would be $1565.6 \times 10^3$ tonnes under SFQP, $1543.2 \times 10^3$ tonnes under BCM, and $1642.5 \times 10^3$ tonnes under WCM; those allocated to the recycling facility would be $1551.3 \times 10^3$ tonnes under SFQP, $1551.3 \times 10^3$ tonnes under BCM, and $1910.7 \times 10^3$ tonnes under WCM. In general, the solutions obtained from BCM and WCM are useful for judging the system’s capability to realize the desired goal; however, the main limitation of the best/worst case analyses is their over-simplification of fuzzy membership information into two extreme values, such that the solutions only under extreme cases can be generated.

Fig. 6. Comparison of waste-flow-allocation patterns among SFQP, BCM and WCM.
4.3. Comparison with stochastic linear programming

Assume that uncertainties in the system components are dependent; the problem can also be formulated as a scenario-based fuzzy-stochastic linear programming (SFLP) model. In SFLP, one membership grade \( \lambda \) is used for representing the satisfaction degree of all system objective and constraints. A SFLP model for the study problem can be formulated as:

\[
\text{Max} \quad \lambda, \\
\text{subject to: } \sum_{t=1}^{T} L_{t}X_{it}(TR_{it} + OP_{it}) + \sum_{t=1}^{T} \sum_{k=1}^{K_{t}} L_{t}p_{ik}Y_{ik}(DR_{it} + DP_{it}) \leq f^{U} - \lambda(f^{U} - f^{L}), \quad k = 1, 2, \ldots, K_{t} \tag{10a}
\]

\[
\sum_{t=1}^{T} L_{t} \left( X_{it} + Y_{i1k} \right) + \sum_{t=1}^{T} \sum_{k=1}^{K_{t}} L_{t}p_{ik}Y_{ik}(DR_{it} + DP_{it}) + FE_{i}(DT_{it} + DP_{it}) - RM_{it} \leq f^{U} - \lambda(f^{U} - f^{L}), \quad k = 1, 2, \ldots, K_{t} \tag{10b}
\]

\[
\sum_{t=1}^{T} L_{t} \left( X_{it} + Y_{i1k} \right) + \sum_{t=1}^{T} \sum_{k=1}^{K_{t}} L_{t}p_{ik}Y_{ik}(DR_{it} + DP_{it}) + FE_{i}(DT_{it} + DP_{it}) - RM_{it} \leq f^{U} - \lambda(f^{U} - f^{L}), \quad k = 1, 2, \ldots, K_{t} \tag{10c}
\]

\[
X_{it} + Y_{i1k} \leq TC_{i}^{U} - \lambda(TC_{i}^{U} - TC_{i}^{L}), \quad \forall t; i = 2, 3; k = 1, 2, \ldots, K_{t} \tag{10d}
\]

\[
\sum_{t=1}^{T} \left( X_{it} + Y_{i1k} \right) = WC_{i1k}^{U} - \lambda(WC_{i1k}^{U} - WC_{i1k}^{L}), \quad \forall t; k = 1, 2, \ldots, K_{t} \tag{10e}
\]

\[
X_{it} + Y_{i1k} \leq DG_{i1k} - \lambda(DG_{i1k} - DG_{i1k}), \quad \forall t; k = 1, 2, \ldots, K_{t} \tag{10f}
\]

\[
A_{i}^{U} \leq X_{it} \leq A_{i}^{L}, \quad \forall i, t \tag{10g}
\]

\[
0 \leq Y_{i1k} \leq X_{i1k}, \quad \forall i, t; k = 1, 2, \ldots, K_{t} \tag{10h}
\]

Fig. 7 provides the solution for waste-flow allocation pattern obtained through the SFLP model, which is also different from that obtained through the SFQP model. Fig. 8 summarizes the comparison result of waste-flow-allocation patterns between SFQP and SFLP. For example, when waste-generation rates are low over the planning horizon, the results from SFQP indicate that the waste flows allocated to the landfill, composting and recycling facilities would be 4871.1 \times 10^{3}, 1565.6 \times 10^{3} and
1551.3 \times 10^3 \text{ tonnes}, respectively; in comparison, the waste flows (from SFLP) to the three facilities would be 4768.0 \times 10^3, 1642.5 \times 10^3 \text{ and } 1564.7 \times 10^3 \text{ tonnes, respectively. When waste-generation rates are high in the three periods (denoted as symbol HHH), the waste flows (from SFQP) allocated to the landfill, composting and recycling facilities would be 5189.5 \times 10^3, 2250.5 \times 10^3 \text{ and } 1631.8 \times 10^3 \text{ tonnes, respectively; the waste flows (from SFLP) to the three facilities would be 5346.5 \times 10^3, 2007.9 \times 10^3 \text{ and } 1703.0 \times 10^3 \text{ tonnes, respectively.}

The above several models mainly based on MSP technique. In MSP, recourse is an ability to take corrective actions after a random event has taken place. The initial action is called the first-stage decision, and the corrective one is named the recourse decision. Thus, the complexity associated with the first-stage variables (i.e. allowable waste-flow levels) is mainly caused by the limited capacity for waste disposal and the increasing waste-generation level in the region. Variations in the values of allowable waste flow would lead to different policies for managing waste disposal and diversion. For example, if the allowable waste flows are regulated at too low levels, then high penalties may have to be paid when the allowances are violated (e.g., particularly when waste-generation rate is high); conversely, if allowable waste flow levels are too high, then high economic losses would be generated (e.g., the planned facility capacities would become too large, leading to much waste of investment cost). Fig. 9 provides the results for the optimized allowable waste flows under these different models.

The optimized allowable waste flows are different from each other. For example, in period 2, the optimized allowable waste flows to the landfill would be 950 t/wk under BCM, 850 t/wk under SFQP, 838.5 t/wk under SFQP-N, 850 t/wk under SFLP, and 821.9 t/wk under WCM. Different policies for allowable waste flows could lead to varied excess flows, and thus lead to changed normal cost and penalties, as shown in Fig. 10. The normal cost for handling allowable waste flows would be $60.7 \times 10^6 \text{ under BCM, } 60.9 \times 10^6 \text{ under SFQP, } 57.7 \times 10^6 \text{ under SFQP-N, } 61.8 \times 10^6 \text{ under SFLP, and } 66.5 \times 10^6 \text{ under WCM; the penalty for treating excess flows would be $5.6 \times 10^6 \text{ under BCM, } 8.7 \times 10^6 \text{ under SFQP, } 12.1 \times 10^6 \text{ under SFQP-N, } 6.7 \times 10^6 \text{ under SFLP, and } 7.2 \times 10^6 \text{ under WCM. The system would encounter the highest penalty under SFQP-N.}

4.4. Cost analysis

Fig. 11 presents the cost values obtained from the several models. The total costs under BCM, SFQP, SFQP-N, SFLP and WCM would be $66.3 \times 10^6, 69.6 \times 10^6, 69.3 \times 10^6, 68.5 \times 10^6 \text{ and } 73.6 \times 10^6 \text{, respectively. Among them, BCM would
Fig. 9. Results for allowable waste flows under different models.

Fig. 10. Normal cost and penalty under different models.

Fig. 11. Total costs obtained from different models.
achieve the lowest total cost since it is based on an anticipation of upper waste-management capacity and lower waste-generation rate; however, it would result in a higher risk of violating system constraints. WCM would have the highest total cost level since it corresponds to a conservative estimation towards the system constraints (i.e. lower facility capacity and upper waste-generation rate), associated with a lower risk level. Generally, decisions at WCM would lead to an increased reliability in fulfilling the system requirements but with a high total cost; decisions at BCM would result in a low total cost, but the risk of violating the system constraints would be high (i.e. a low reliability level of satisfying system constraints). In comparison, the total cost from SFQP would lie within the solutions of BCM and WCM. It demonstrates that SFQP’s results can provide more useful information for making compromises among waste-management cost, environmental requirement, and constraint-violation risk. The total cost from SFLP would slightly lower than that from SFQP. This is mainly due to the fact that the majority of $\lambda_i$ (for system constraints in SFQP) would approach zero. According to the SFQP modeling formulation, when $\lambda_i > 0$, the boundary of constraint $i$ would be moved inward closer to $b_i$ (i.e. approaching constraint’s lower bound); when $\lambda_i < 0$, this boundary would be moved outward closer to $b_i + P_i$ (i.e. approaching constraint’s upper bound); when $\lambda_i = 0$, this boundary would be closer to $b_i + P_i/2$ (i.e. constraint’s medium value). In comparison, the solution of $\lambda$ value from SFLP would be 0.53, implying that the waste treated is less than the medium value of waste-generation amount; lower waste-disposal amount could result in a lower total cost. Moreover, the major problem with the SFLP method lies in its using one $\lambda$ grade for representing the satisfaction degree of all system objective and constraints, such that some constraints may not be well satisfied while some others may be over-satisfied. This would then lead to a low effectiveness in accomplishing the objective and satisfying the constraints. In comparison, multiple membership grades (multiple $\lambda_i$ levels) as adopted in SFQP can tackle the uncertainties in the model’s right-hand sides independently, such that can effectively optimize the general satisfaction of the objective and constraints.

5. Conclusions

In this study, a scenario-based fuzzy-stochastic quadratic programming (SFQP) method has been developed for planning municipal solid waste (MSW) management systems under uncertainty. The developed SFQP incorporates fuzzy quadratic programming (FQP) and scenario-based multistage stochastic programming (MSP) within a general framework; this allows dual uncertainties presented as probability distributions and fuzzy sets being communicated into the optimization process. The developed SFQP method has been applied to a case study of long-term MSW management and planning. Dynamics and uncertainties of waste generation and allocation could be taken into account through a multilayer scenario tree. Moreover, various scenarios that are associated with different levels of economic penalties when the pre-regulated policy targets are violated have been analyzed. Sensitivity analysis has also been conducted to reflect the effects of varied waste-management capacities on the system cost and the satisfaction degree under SFQP and SFQP-N. The solutions obtained can be used to generate multiple decision alternatives, such that desired policies under various environmental, economic, and system-reliability conditions can eventually be identified.

Comparisons of the developed SFQP with BCM, WCM and FSLP models have also been conducted. SFQP has advantages in reflecting the complexities of uncertainties presented as fuzzy sets and probabilistic distributions as well as their combinations. The results obtained from SFQP can help to identify desired waste-flow-allocation plans to make a compromise among system cost, satisfaction degree, and constraint-violation risk. Multiple membership grades (multiple $\lambda_i$ levels) as adopted in SFQP can tackle the uncertainties in the model’s right-hand sides independently, such that can effectively optimize the general satisfaction of the objective and constraints. Although this study is the first attempt for identifying optimal waste management policies through the SFQP approach, the results suggest that the developed method is applicable for other environmental and resources planning problems (e.g., air quality management and water resources planning) that are associated with dynamic complexities within multistage contexts as well as uncertainties in multiple formats.

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References