An integrated two-stage optimization model for the development of long-term waste-management strategies

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ABSTRACT

In this study, an integrated two-stage optimization model (ITOM) is developed for the planning of municipal solid waste (MSW) management in the City of Regina, Canada. The ITOM improves upon the existing optimization approaches with advantages in uncertainty reflection, dynamic analysis, policy investigation, and risk assessment. It can help analyze various policy scenarios that are associated with different levels of economic penalties when the promised policy targets are violated, and address issues concerning planning for a cost-effective diversion program that targets on the prolongation of the existing landfill. Moreover, violations for capacity and diversion constraints are allowed under a range of significance levels, which reflect the tradeoffs between system-cost and constraint-violation risk. The modeling results are useful for generating a range of decision alternatives under various environmental, socio-economic, and system-reliability conditions. They are valuable for supporting the adjustment (or justification) of the existing waste-management practices, the long-term capacity planning for the city’s waste-management system, and the identification of desired policies regarding waste generation and management.

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1. Introduction

The conventional waste collection and disposal system in North America consists of garbage trucks and a landfill. However, the trend for disposal of municipal solid waste (MSW) is towards implementing waste diversion and creating an integrated MSW management system (Environment Canada, 2000; Chang et al., 2005). Thus, the conventional single-choice management, i.e. reliance on landfill for waste disposal, is inadequate. Complexities exist in such a diversion effort, including collection techniques to be used, levels of service to be offered, and facilities to be adopted; moreover, many related processes and/or factors are complex with interactive, dynamic and uncertain features. Therefore, a systems approach for analyzing waste management is desired, in order to support decisions of short-term waste-management operation and long-term strategic planning.

A large number of deterministic optimization techniques have been developed for supporting decisions of MSW management and evaluating relevant operation and investment policies since the 1960s (Anderson, 1968); they involved linear, dynamic, integer and multi-objective programming methods, as well as a number of cities in the world. For example, Kirca and Erkip (1988) formulated a linear programming model for determining transfer station locations in the MSW management system of Istanbul, Turkey. Or and Curi (1993) applied a mixed integer linear programming model for improving solid waste collection and transportation system in
the City of Izmir, Turkey, in order to minimize the city’s total solid waste collection and transportation costs. Chang et al. (1997) explored the cost-effective and workload-balancing operation in the regional solid waste-management system of the Taipei City, where linear and integer programming methods were applied within a two-stage analytical framework. More recently, Fiorucci et al. (2003) developed a decision-support system (DSS) for solid waste-management planning in the City of Genova, Italy, where waste-management-facility numbers as well as waste-flow patterns were examined, with the objective of minimizing the recycling, transportation and maintenance costs. Chang et al. (2005) utilized integer programming method to support the decisions of location and capacity for a material-recovery facility in the City of San Antonio, Texas, USA.

However, in the real-world MSW management problems, uncertainties may exist in the related costs, impact factors and objectives, which will affect the optimization processes and the decision schemes generated (Huang et al., 1992). The complexities could be further compounded not only by interactions among the uncertain parameters but also through additional economic implications. Such complexities have placed many MSW management problems beyond deterministic programming methods. Various methods dealing with uncertainties were developed for MSW management and planning, such as fuzzy, stochastic and interval mathematical programming (abbreviated as FMP, SMP and IMP, respectively). For example, Koo et al. (1991) proposed a framework using WRAP (Waste Resources Allocation Program) and fuzzy set theory to address tradeoffs among the objectives of economic efficiency, environmental quality, and administrative efficiency, such that an optimal site for a new hazardous waste treatment facility in southwestern Korea could be determined. Arey et al. (1993) applied a mixed optimization and probabilistic-analysis approach for determining daily waste-management practices in the Municipalities of Hamilton and St. Catharines, Ontario, Canada. Huang et al. (1997, 1998) proposed interval-parameter programming techniques for planning solid waste management and capacity expansion in the Regional Municipality of Hamilton-Wentworth, Ontario, Canada; the developed techniques could readily process interval data, thereby avoiding problems encountered by other optimization methods when faced with complexities in parameter uncertainties and solution algorithms. Chen and Chang (2000) formulated a grey fuzzy dynamic model for the prediction of solid waste generation in the City of Tainan, Taiwan, where the technique of fuzzy goal regression was employed to minimize the discrepancy between the predicted and observed values. More recently, Davila et al. (2005) proposed a grey integer programming-based game theory (GIP-based game theory) for system optimization and cost-benefit analysis at two competing landfills in the Lower Rio Grande Valley, Texas, USA.

Although many studies of waste management under uncertainty were conducted for a number of cities in the world, there have been no previous studies focusing on proposing an integrated two-stage optimization model (ITOM) for a real study of MSW management under uncertainty. Therefore, the objective of this study is to develop such an ITOM for the planning of MSW management in the City of Regina, Canada. In ITOM, approaches of stochastic mathematical programming (SMP) and interval mathematical programming (IMP) will be incorporated within an integer programming (IP) framework for accounting for multiple uncertainties, the relevant economic penalties, system reliabilities, and capacity-expansion decision issues.

2. Methodology

Two-stage stochastic programming (TSP) method is effective for problems where an analysis of policy scenarios is desired and the related data are mostly uncertain. In TSP, decision variables are divided into two subsets: those that must be determined before random variables are disclosed, and those (recourse variables) that will be determined after the uncertainties are disclosed. A general TSP model can be formulated as follows (Birge and Louveaux, 1997):

\[
\begin{align*}
z &= \min \ C^T X + \mathbb{E}_{\omega}(Q(X, \omega)) \\
\text{subject to:} \quad &x = X \\
&Q(x, \omega) = \min f(\omega)^T y \\
\text{subject to:} \quad &D(\omega)y \geq h(\omega) + T(\omega)x \\
&y \in Y
\end{align*}
\]

where \( X \subset \mathbb{R}^x, C \subset \mathbb{R}^x \), and \( Y \subset \mathbb{R}^x \). Here, \( \omega \) is a random variable from space \( (\Omega, F, P) \) with \( \Omega \subset \mathbb{R}^r, f : \Omega \rightarrow \mathbb{R}^r, h : \Omega \rightarrow \mathbb{R}^m, D : \Omega \rightarrow \mathbb{R}^{m \times x}, \) and \( T : \Omega \rightarrow \mathbb{R}^{m \times x} \). By letting random variables (i.e. \( \omega \)) take discrete values \( \omega_i \) with probability levels \( p_n(h=1, 2, ..., v) \) and \( \sum p_n = 1 \), the above TSP can be equivalently formulated as a linear programming model as follows (Ahmed et al., 2004; Li et al., 2007):

\[
\begin{align}
\text{Min } f &= C^T X + \sum_{h=1}^{v} p_b D_i Y \\
\text{subject to:} \\
A_1 X &\leq B_1, \quad r = 1, 2, ..., m_1 \\
A_2 X + A_2^T Y &\geq w_b, \quad t = 1, 2, ..., m_2; h = 1, 2, ..., v \\
x_j &\geq 0, \quad x_j \in X, \quad j = 1, 2, ..., n_1 \\
y_{jh} &\geq 0, \quad y_{jh} \in Y, \quad j = 1, 2, ..., n_2; h = 1, 2, ..., v.
\end{align}
\]

Obviously, model (2) can deal with uncertainties in the right-hand sides presented as probability distributions when coefficients in the left-hand sides and in the objective function are deterministic. However, in real-world optimization problems, the quality of information that can be obtained is mostly not satisfactory enough to be presented as probabilities (Li et al., 2006). Such complexities cannot be solved through model (2). IMP is effective in tackling uncertainties expressed as interval values with known lower and upper bounds.

\section{Acknowledgments}

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bounds but unknown distribution functions (Huang et al., 1992). Therefore, through incorporating IMP and TSP within a general optimization framework, a hybrid two-stage programming linear model can be formulated as follows:

Min \( f^c = C_i^c X^c + \sum_{h=1}^{v} p_i D_i^c Y^c \)  
subject to:
\( A_i^c X^c \leq B_i, \quad r = 1, 2, \ldots, m_1 \) \hspace{1cm} (3a)
\( A_i^{c^t} X^c + A_i^{c^t Y} \geq w_i^c, \quad t = 1, 2, \ldots, m_2; h = 1, 2, \ldots, v \) \hspace{1cm} (3b)
\( x_j^c \geq 0, \quad x_j^c \in X^c; j = 1, 2, \ldots, n_1 \) \hspace{1cm} (3c)
\( y_{jh}^c \geq 0, \quad y_{jh}^c \in Y^c; j = 1, 2, \ldots, n_2; h = 1, 2, \ldots, v \) \hspace{1cm} (3d)

where \( A_i \in (R^x)^{m_x, n_x}, A_i \in (R^x)^{m_x, n_y}, B_i \in (R^x)^{m_x, 1}, C_i \in (R^x)^{1, n_x}, D_i \in (R^y)^{1, n_y}, X \in (R^x)^{n_x}, Y \in (R^y)^{n_y} \) and \([R]\) denote a set of interval parameters and/or variables; superscripts ‘‘c’’ and ‘‘c^t’’ represent lower and upper bounds of the interval values, respectively. In a real-world MSW management problem, randomness in other right-hand-side parameters such as available waste-management capacities, also need to be reflected. For example, the waste-management capacity can be fixed with a level of probability, i.e. \( q_i \in [0, 1] \) (significance level), which represents the admissible risk of violating the uncertain capacity constraint. The chance-constrained programming (CCP) method can be used for dealing with this type of uncertainty. In CCP, it is required that the constraints be satisfied under given probabilities. A general CCP formulation can be expressed as (Charnes et al., 1972):

Min \( c^T x \)  
subject to:
\( P_r \{ A_i(t) X^c \leq b_i(t) \} \geq 1 - q_i, A_i(t) = A(t), s = 1, 2, \ldots, m \) \hspace{1cm} (4b)
\( x \geq 0. \) \hspace{1cm} (4c)

Model (4) can be converted into deterministic versions through: (i) fixing a certain level of probability \( q_i \) for uncertain constraint \( s \), and (ii) imposing the condition that the constraint should be satisfied with at least a probability level of 1–\( q_i \) (Huang, 1998). Therefore, to deal with uncertainties presented in terms of interval values and random variables and to reflect the reliability of satisfying (or risk of violating) system constraints, the approaches of TSP, IMP and CCP can be incorporated within an integer programming (IP) framework. This leads to an integrated two-stage optimization model (ITOM) as follows:

Min \( f^c = C_i^c X^c + \sum_{h=1}^{v} p_i D_i^c Y^c \)  
subject to:
\( A_i^c X^c \leq B_i, \quad r = 1, 2, \ldots, m_1 \) \hspace{1cm} (5a)
\( A_i^{c^t} X^c + A_i^{c^t Y} \geq w_i^c, \quad t = 1, 2, \ldots, m_2; h = 1, 2, \ldots, v \) \hspace{1cm} (5b)
\( x_j^c \geq 0, \quad x_j^c \in X^c; j = 1, 2, \ldots, n_1 \) \hspace{1cm} (5c)
\( y_{jh}^c \geq 0, \quad y_{jh}^c \in Y^c; j = 1, 2, \ldots, n_2; h = 1, 2, \ldots, v \) \hspace{1cm} (5d)

In model (5), decision variables can be sorted into two deterministic submodels that correspond to the lower and upper bounds of desired objective function value. This transformation process is based on an interactive algorithm, which is different from the best/worst case analysis (Huang et al., 1992). Interval solutions associated with varying levels of constraint-violation risk can then be obtained by solving the two submodels sequentially. The submodel corresponding to the lower-bound objective function value \( f^- \) can be firstly formulated as follows (assume that \( B^+ > 0 \) and \( f^+ = 0 \)):

Min \( f^c = \sum_{j=1}^{k_1} c_j^c x_j^c + \sum_{j=k_1+1}^{n_1} c_j^c x_j^c + \sum_{h=1}^{v} p_h d_j^c y_{jh}^c \)  
subject to:
\( \sum_{j=1}^{k_1} |a_{1j}| \cdot \text{Sign}(a_{1j}^c) x_j^c + \sum_{j=k_1+1}^{n_1} |a_{1j}| \cdot \text{Sign}(a_{1j}^c) x_j^c \leq b_j^c, \forall r \) \hspace{1cm} (6b)
\( \sum_{j=1}^{k_1} |a_{2j}| \cdot \text{Sign}(a_{2j}^c) x_j^c + \sum_{j=k_1+1}^{n_1} |a_{2j}| \cdot \text{Sign}(a_{2j}^c) x_j^c + \sum_{j=1}^{n_1} |a_{2j}| \cdot \text{Sign}(a_{2j}^c) y_{jh}^c + \sum_{j=k_1+1}^{n_1} |a_{2j}| \cdot \text{Sign}(a_{2j}^c) y_{jh}^c \geq w_j^c, \forall r, h \) \hspace{1cm} (6c)
\( \sum_{j=1}^{k_1} |a_{3j}| \cdot \text{Sign}(a_{3j}^c) x_j^c + \sum_{j=k_1+1}^{n_1} |a_{3j}| \cdot \text{Sign}(a_{3j}^c) x_j^c + \sum_{j=1}^{n_1} |a_{3j}| \cdot \text{Sign}(a_{3j}^c) y_{jh}^c + \sum_{j=k_1+1}^{n_1} |a_{3j}| \cdot \text{Sign}(a_{3j}^c) y_{jh}^c \leq (b_j^c)^{+}, \forall r, h \) \hspace{1cm} (6d)
\( x_j^c \geq 0, \quad j = 1, 2, \ldots, k_1 \) \hspace{1cm} (6e)
\( x_j^c \geq 0, \quad j = k_1 + 1, k_1 + 2, \ldots, n_1 \) \hspace{1cm} (6f)
\( y_{jh}^c \geq 0, \forall h; j = 1, 2, \ldots, k_2 \) \hspace{1cm} (6g)
\( y_{jh}^c \geq 0, \forall h; j = k_2 + 1, k_2 + 2, \ldots, n_2 \) \hspace{1cm} (6h)

where \( x_j^c, j = 1, 2, \ldots, k_1, k_1 + 1, k_1 + 2, \ldots, n_1, n_1 \), are interval variables with positive coefficients in the objective function; \( x_j^c, j = k_1 + 1, k_1 + 2, \ldots, n_1 \), are interval variables with negative coefficients; \( y_{jh}^c, j = 1, 2, \ldots, k_2, n_1 \), are interval variables with positive coefficients in the objective function; \( y_{jh}^c, j = k_2 + 1, k_2 + 2, \ldots, n_2 \), \( n_2 \), and \( h = 1, 2, \ldots, v \) are random variables with positive coefficients. Solutions of \( x_{1j}^{\text{opt}}(j = 1, 2, \ldots, k_1), x_{2j}^{\text{opt}}(j = k_1 + 1, k_1 + 2, \ldots, n_1) \), and \( y_{jh}^{\text{opt}}(j = 1, 2, \ldots, k_2) \), \( y_{jh}^{\text{opt}}(j = k_2 + 1, k_2 + 2, \ldots, n_2) \) can be obtained through submodel (6). Based on the above solutions, the second submodel for \( f^+ \) can be formulated as follows:

Min \( f^c = \sum_{j=1}^{k_1} c_j^c x_j^c + \sum_{j=k_1+1}^{n_1} c_j^c x_j^c + \sum_{h=1}^{v} p_h d_j^c y_{jh}^c \)  
subject to:
\( \sum_{j=1}^{k_1} c_j^c x_j^c + \sum_{j=k_1+1}^{n_1} c_j^c x_j^c + \sum_{h=1}^{v} p_h d_j^c y_{jh}^c \) \hspace{1cm} (7a)

subject to:
\[
\begin{align*}
\sum_{j=1}^{k_0} a_{0j} x_{0j} + \sum_{j=k_0+1}^{n_1} a_{0j} x_{0j} + \sum_{j=k_0+1}^{n_2} a_{0j} x_{0j} & \leq b_{vj}, \forall r, t \quad (7b) \\
\sum_{j=1}^{k_1} a_{1j} x_{1j} + \sum_{j=k_1+1}^{n_3} a_{1j} x_{1j} + \sum_{j=k_1+1}^{n_4} a_{1j} x_{1j} & \leq b_{vj}, \forall t, h \quad (7c) \\
\sum_{j=1}^{k_2} a_{2j} x_{2j} + \sum_{j=k_2+1}^{n_5} a_{2j} x_{2j} + \sum_{j=k_2+1}^{n_6} a_{2j} x_{2j} & \leq b_{vj}, \forall s, h \quad (7d)
\end{align*}
\]

\[x_{0j}^i \geq x_{0j}^{*\text{opt}}, j = 1, 2, \ldots, k_1 \quad (7e)\]
\[0 \leq x_{0j}^i \leq x_{0j}^{+\text{opt}}, j = k_1 + 1, k_1 + 2, \ldots, n_1 \quad (7f)\]
\[y_{1j}^i \geq y_{1j}^{*\text{opt}}, \forall h, j = 1, 2, \ldots, k_2 \quad (7g)\]
\[0 \leq y_{1j}^i \leq y_{1j}^{+\text{opt}}, \forall h, j = k_2 + 1, k_2 + 2, \ldots, n_2 \quad (7h)\]

Solutions of \(x_{0j}^{*\text{opt}}\) and \(y_{1j}^{*\text{opt}}\) can be obtained through submodel (6) and (7), interval solution for model (5) under a set of \(q_s, s = 1, 2, \ldots, m_s\) based on further information of waste-generation amounts and/or fines caused by the improper policies. Thus, the plan for allowable waste flows is presented (i.e., excess flow = generated flow - assigned quota). The surplus flow will be disposed of at a premium, resulting in excess costs (penalties) to the system. The penalties are associated with operating cost for excess waste flows as shipped locally to alternative and more expensive facilities, collection and transportation costs for excess flows to more remote facilities, and extra expenses and/or fines caused by the improper policies. Thus, the plan for allowable waste-flow allocation at the beginning is named the first-stage decision, and that for the recourse actions is called the second-stage decision. The first-stage decision has to be made before further waste-generation information is disclosed, whereas the second-stage one is to adapt to the previous decision based on further information of waste-generation levels. Under such a situation, the total waste flow will be the sum of both fixed allowable and probabilistic surplus flows.

Table 3 presents the waste-generation rates and the associated probabilities of occurrence in the five planning periods, indicating that the waste-generation amounts are highly uncertain, presented by intervals with the associated probabilities varying dynamically. Table 4 shows the capacity-expansion options and the relevant capital costs for different waste-management facilities. For example, for the landfill development issue, there are three options being taken into account by the city: (a) developing a new landfill at the southern, (b) developing a new landfill at the north, and (c) expanding the existing landfill. Moreover, the Regina Round Table on Solid Waste Management (RRTSWM) developed a 20-year solid waste-management strategy (City of Regina, 2005). The city is putting efforts on increasing waste-diversion rate through developing an integrated solid waste-management approach.

Generally, many challenging questions will arise in the study system, such that (i) what is the least-cost means for meeting the reduction and/or diversion goals? (ii) if a new landfill is needed, where should it be located? (iii) what new facilities should be

3. **The study system**

The City of Regina is the capital of the Province of Saskatchewan, and is located in the heart of western Canada. Its population is about 195,000, where the households generate residential wastes of 71,000 to 82,000 tonnes/year (Statistics Canada, 2006; City of Regina, 2006). Solid waste management in the city covers many areas, ranging from garbage collection to environmental protection. It involves the provision of specific and personal services to most of the residents (through waste collection and recovery) as well as indirect services to the entire community (through waste disposal and recovery). The generated solid wastes typically include paper, yard waste, food waste, plastics, metals, glass, wood and other items. Consistent with many communities in western Canada, the city relies mostly on a sanitary landfill for disposing of its MSW. The landfill is located in the northeastern part of the city, occupying 97 ha with an actual landfilling area of 60 ha. Approximately 65,000 tonnes/year of MSW generated from the residential sector were buried at the landfill (nearly 90% of the total waste generated by households). The existing landfill is expected to be able to accept waste till 2011 or 2012 (City of Regina, 2005). Due to the scarcity of land around the urban center and the growing opposition from the public with regard to landfill operation, the city is making efforts on waste diversion through the integrated solid waste-management (ISWM) approach to change the current practice of relying solely on landfill for its waste disposal.

The study time horizon is 25 years (from 2006 to 2030), which is further divided into five 5-year periods. Tables 1 and 2 contain regular collection and transportation costs for allowable waste flows, operating costs for waste-management facilities, penalties for surplus waste flows, and revenues from recycling and composting facilities over the five planning periods. A projected allowable waste-flow level (quota of waste flow) to each facility is pre-regulated by the Municipal Engineering Department of the City of Regina. If this level is not exceeded, a regular (normal) cost will apply to the system. However, if it is exceeded, an excess waste flow will be generated (i.e., excess flow = generated flow - assigned quota). The surplus flow will be disposed of at a premium, resulting in excess costs (penalties) to the system. The penalties are associated with (i) operating cost for excess waste flows as shipped locally to alternative and more expensive facilities, (ii) collection and transportation costs for excess flows to more remote facilities, and (iii) extra expenses and/or fines caused by the improper policies. Thus, the plan for allowable waste-flow allocation at the beginning is named the first-stage decision, and that for the recourse actions is called the second-stage decision. The first-stage decision has to be made before further waste-generation information is disclosed, whereas the second-stage one is to adapt to the previous decision based on further information of waste-generation levels. Under such a situation, the total waste flow will be the sum of both fixed allowable and probabilistic surplus flows.

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Generally, many challenging questions will arise in the study system, such that (i) what is the least-cost means for meeting the reduction and/or diversion goals? (ii) if a new landfill is needed, where should it be located? (iii) what new facilities should be
flows? Thus, the proposed method will be applied to the city’s long-term solid waste-management planning. Issues concerning planning for cost-effective diversion and prolongation of the landfill will be addressed. The modeling results will be used for supporting decisions of the city’s long-term MSW management, such as (a) identification of desired capacity-expansion schemes for waste-management-facilities, (b) allocation of waste flows to suitable facilities, and (c) analysis of the tradeoff between the cost of waste management and the risk of system disruption.

The modeling formulation (based on the developed ITOM method) for the city’s MSW management is presented in the Appendix. The model includes continuous and binary decision variables. The binary variables represent the development or expansion options for waste-management-facilities in different periods (i.e. $Y_{ik}$ and $Z_{ik}$); their solutions can be used for answering the questions related to timing, sizing and siting for waste-management-facility development and/or expansion under uncertainty. The continuous variables represent the optimized waste flows from the city to the waste-management-facilities. Furthermore, the continuous variables include two subsets: those (the first-stage ones, $T_{ik}$) that must be determined before the random variables (i.e. waste-generation rates) are disclosed, and those (the second-stage ones, $X_{ik}$) that will be determined after the random variables are disclosed. A set of chance constraints on waste-management capacities and waste-diversion rates are considered, which can help investigate the risks of violating the capacity and diversion constraints. Interval solutions under given significance levels can be obtained based on solution method as described in Section 2. They will be useful for generating a range of decision alternatives under various environmental, socioeconomic, and system-reliability conditions.

### 4. Result and discussion

In this study, a set of chance constraints on waste-management capacities and waste-diversion rates are considered, which can help investigate the risks of violating the capacity and diversion constraints. Table 5 presents the solutions of facility expansion schemes under different $q_i$ levels (i.e. significance levels, which represent the admissible levels of constraint-violation risk). Under $q_i=0.01$, 0.05 and 0.10, the results indicate that the landfill

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**Table 1 – Costs and revenues for allowable waste flows**

<table>
<thead>
<tr>
<th>Planning period</th>
<th>$k=1$</th>
<th>$k=2$</th>
<th>$k=3$</th>
<th>$k=4$</th>
<th>$k=5$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Operating costs of waste-management-facilities, $OP_{ik}$ ($/tonne$):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Landfill</td>
<td>[9, 17]</td>
<td>[6.9, 13.02]</td>
<td>[5.83, 11.02]</td>
<td>[4.94, 9.32]</td>
<td>[4.18, 7.89]</td>
</tr>
<tr>
<td>Composting</td>
<td>[21, 26]</td>
<td>[16.09, 19.92]</td>
<td>[13.61, 16.85]</td>
<td>[11.52, 14.26]</td>
<td>[9.74, 12.06]</td>
</tr>
<tr>
<td>Recycling</td>
<td>[61, 67.8]</td>
<td>[46.74, 51.95]</td>
<td>[39.54, 43.95]</td>
<td>[33.45, 37.18]</td>
<td>[28.30, 31.46]</td>
</tr>
<tr>
<td><strong>Collection and transportation costs, $TR_{ik}$ ($/tonne$):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>To landfill</td>
<td>[32, 37]</td>
<td>[24.52, 28.35]</td>
<td>[20.74, 23.98]</td>
<td>[17.55, 20.29]</td>
<td>[14.85, 17.17]</td>
</tr>
<tr>
<td>To composting</td>
<td>73</td>
<td>55.93</td>
<td>47.32</td>
<td>40.03</td>
<td>33.87</td>
</tr>
<tr>
<td>To recycling</td>
<td>101</td>
<td>77.38</td>
<td>65.47</td>
<td>55.39</td>
<td>46.86</td>
</tr>
<tr>
<td><strong>Revenues from waste-management-facilities, $RE_{ik}$ ($/tonne$):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composting</td>
<td>[5.0, 10.0]</td>
<td>[3.83, 6.69]</td>
<td>[3.24, 4.48]</td>
<td>[2.74, 5.48]</td>
<td>[2.32, 6.64]</td>
</tr>
<tr>
<td>Recycling</td>
<td>[45.0, 55.0]</td>
<td>[34.48, 42.14]</td>
<td>[29.17, 35.65]</td>
<td>[24.68, 30.16]</td>
<td>[20.88, 25.52]</td>
</tr>
<tr>
<td><strong>Costs of residue disposal at the landfill, $FT_{ik}$ ($/tonne$):</strong></td>
<td></td>
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<tr>
<td>From composting</td>
<td>[1.68, 2.1]</td>
<td>[1.287, 1.63]</td>
<td>[1.089, 1.36]</td>
<td>[0.92, 1.152]</td>
<td>[0.779, 0.974]</td>
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<tr>
<td>From recycling</td>
<td>[1.47, 1.68]</td>
<td>[1.126, 1.287]</td>
<td>[0.953, 1.089]</td>
<td>[0.806, 0.92]</td>
<td>[0.68, 0.779]</td>
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**Table 2 – Costs and revenues for excess waste flows**

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<th>$k=2$</th>
<th>$k=3$</th>
<th>$k=4$</th>
<th>$k=5$</th>
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<tr>
<td><strong>Operating costs of waste-management-facilities, $DP_{ik}$ ($/tonne$):</strong></td>
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<tr>
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<td>[11.67, 22.04]</td>
<td>[9.87, 16.09]</td>
<td>[8.35, 15.78]</td>
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<tr>
<td>Composting</td>
<td>[34, 42]</td>
<td>[26.05, 32.18]</td>
<td>[22.04, 27.23]</td>
<td>[18.65, 23.03]</td>
<td>[15.78, 19.49]</td>
</tr>
<tr>
<td>Recycling</td>
<td>[104, 115.3]</td>
<td>[79.69, 88.34]</td>
<td>[67.41, 74.74]</td>
<td>[57.04, 62.33]</td>
<td>[48.25, 55.50]</td>
</tr>
<tr>
<td><strong>Collection and transportation costs, $DR_{ik}$ ($/tonne$):</strong></td>
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<tr>
<td>To landfill</td>
<td>[48, 55.5]</td>
<td>[36.77, 42.52]</td>
<td>[31.11, 35.98]</td>
<td>[26.32, 30.27]</td>
<td>[22.27, 25.75]</td>
</tr>
<tr>
<td>To composting</td>
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<td>83.89</td>
<td>70.98</td>
<td>60.05</td>
<td>50.81</td>
</tr>
<tr>
<td>To recycling</td>
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<td>116.07</td>
<td>98.20</td>
<td>83.08</td>
<td>70.29</td>
</tr>
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<td><strong>Revenues from waste-management-facilities, $RM_{ik}$ ($/tonne$):</strong></td>
<td></td>
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<td></td>
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<tr>
<td>Composting</td>
<td>[5.0, 10.0]</td>
<td>[3.83, 6.69]</td>
<td>[3.24, 4.48]</td>
<td>[2.74, 5.48]</td>
<td>[2.32, 6.64]</td>
</tr>
<tr>
<td>Recycling</td>
<td>[45.0, 55.0]</td>
<td>[34.48, 42.14]</td>
<td>[29.17, 35.65]</td>
<td>[24.68, 30.16]</td>
<td>[20.88, 25.52]</td>
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<tr>
<td><strong>Costs of residue disposal at the landfill, $DT_{ik}$ ($/tonne$):</strong></td>
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</tr>
<tr>
<td>From composting</td>
<td>[2.52, 3.15]</td>
<td>[1.93, 2.41]</td>
<td>[1.63, 2.04]</td>
<td>[1.38, 1.73]</td>
<td>[1.17, 1.46]</td>
</tr>
<tr>
<td>From recycling</td>
<td>[2.21, 2.52]</td>
<td>[1.69, 1.93]</td>
<td>[1.43, 1.63]</td>
<td>[1.21, 1.38]</td>
<td>[1.03, 1.17]</td>
</tr>
</tbody>
</table>
would be expanded (based on the existing landfill) at the start of period 2 (the start of 2010) with an incremental capacity of 71.3 ha (i.e. \( Y_{12, \text{opt}} = [1, 1] \)). However, only 28.5 ha (approximately 40% of the total) would be dedicated to the city’s residential waste, since the IC&I (industrial, commercial and institutional) capacity for the composting facility would be 763, 567 and 189 tonnes/week at the start of period 1, with 70% of the capacity (i.e. 270 tonnes/week) being dedicated to the residential waste; then this facility should be expanded by an increment of 189 tonnes/week at the start of periods 3 and 4, respectively (i.e. \( Z_{314, \text{opt}} = [1, 1] \)). When \( q=0.05 \) and 0.10, a centralized composting facility would be developed with a capacity of 189 tonnes/week at the start of period 1; then this facility should be expanded by an increment of 189 tonnes/week at the starts of periods 3 and 4, respectively. Consequently, the expanded capacities for the composting facility would be 763, 567 and 567 tonnes/week under \( q=0.01, 0.05 \) and 0.10, respectively. The recycling facility would be expanded twice (each with an increment of 350 tonnes/week) at the starts of periods 1 and 3 when \( q=0.01, 0.05 \) and 0.10 (i.e. \( Z_{11, \text{opt}} = [1, 1] \) and \( Z_{13, \text{opt}} = [1, 1] \)); then this facility would be expanded by 350 tonnes/week at the start of period 4 when \( q=0.01 \) and 0.05 (i.e. \( Z_{214, \text{opt}} = [1, 1] \)), and expanded by 350 tonnes/week at the start of period 5 when \( q=0.10 \) (\( Z_{215, \text{opt}} = [1, 1] \)).

Generally, an increased \( q \) level means a raised risk of violating the modeling constraints but, at the same time, it leads to a decreased strictness for the capacity constraints and thus results in a lower capacity-expansion amount and a lower capital cost, and vice versa. Table 5 also provide the results for expanded capacity of composting and recycling facilities being 861 and 1050 tonnes/week when \( q=0 \), further demonstrating that more diversion capacity increments would be needed if no constraint violation is allowed.

Table 6 shows the solutions of waste-flow allocation during periods 1 to 5 under different \( q \) levels. An excess flow will be generated if the allowable-waste-flow level is exceeded (i.e. excess flow=generated flow−assigned quota). The waste-flow patterns (including allowable and excess flows) would vary dynamically due to temporal and spatial variations of waste-generation/management conditions. Analyses of the flows allocated to the landfilling, composting and recycling facilities (when the waste-generation rate is high in period 1) indicate:

(a) for the landfill, the optimized allowed flows would be [1068, 1093] tonnes/week; the optimized excess flows would be [146.6, 159.6] tonnes/week under \( q=0.01, \)

### Table 4 – Capacity-expansion options and the relevant capital costs

<table>
<thead>
<tr>
<th>Expansion option</th>
<th>Total expanded capacity</th>
<th>Expanded capacity for residential</th>
<th>Expansion cost (( \text{$10}^6 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Landfill (( \Delta \text{LC} ))</strong>:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansion (( i=1 ))</td>
<td>71.3 ha</td>
<td>28.5 ha</td>
<td>6.32</td>
</tr>
<tr>
<td>North site (( i=2 ))</td>
<td>71.3 ha</td>
<td>28.5 ha</td>
<td>[9.60, 10.45]</td>
</tr>
<tr>
<td>South site (( i=3 ))</td>
<td>71.3 ha</td>
<td>28.5 ha</td>
<td>[6.44, 14.94]</td>
</tr>
<tr>
<td><strong>Composting facility (( \Delta \text{TC}_{\text{com}} ))</strong>:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Option 1 (( m=1 ))</td>
<td>189 tonnes/week</td>
<td>122 tonnes/week</td>
<td>[2.12, 2.54]</td>
</tr>
<tr>
<td>Option 2 (( m=2 ))</td>
<td>385 tonnes/week</td>
<td>270 tonnes/week</td>
<td>[4.23, 5.08]</td>
</tr>
<tr>
<td>Option 3 (( m=3 ))</td>
<td>483 tonnes/week</td>
<td>338 tonnes/week</td>
<td>[5.29, 6.35]</td>
</tr>
<tr>
<td><strong>Recycling facility (( \Delta \text{TC}_{\text{rec}} ))</strong>:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Option 1 (( m=1 ))</td>
<td>350 tonnes/week</td>
<td>140 tonnes/week</td>
<td>[4.51, 5.00]</td>
</tr>
<tr>
<td>Option 2 (( m=2 ))</td>
<td>700 tonnes/week</td>
<td>280 tonnes/week</td>
<td>[9.02, 10.01]</td>
</tr>
<tr>
<td>Option 3 (( m=3 ))</td>
<td>875 tonnes/week</td>
<td>350 tonnes/week</td>
<td>[10.86, 12.51]</td>
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</table>

### Table 5 – Solutions of capacity development/expansion under different \( q \) levels

<table>
<thead>
<tr>
<th>Facility</th>
<th>Symbol</th>
<th>Total expanded capacity</th>
<th>Expanded capacity for residential</th>
<th>Period</th>
<th>Capacity-expansion solution $q=0$</th>
<th>$q=0.01$</th>
<th>$q=0.05$</th>
<th>$q=0.10$</th>
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</thead>
<tbody>
<tr>
<td>Landfill</td>
<td>( Y_{12, \text{opt}} )</td>
<td>71.3 ha</td>
<td>28.5 ha</td>
<td>2</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>Composting</td>
<td>( Z_{411, \text{opt}} )</td>
<td>189 tonnes/week</td>
<td>132 tonnes/week</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td></td>
<td>( Z_{421, \text{opt}} )</td>
<td>385 tonnes/week</td>
<td>270 tonnes/week</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>[1, 1]</td>
</tr>
<tr>
<td></td>
<td>( Z_{431, \text{opt}} )</td>
<td>483 tonnes/week</td>
<td>338 tonnes/week</td>
<td>1</td>
<td>[1, 1]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( Z_{413, \text{opt}} )</td>
<td>189 tonnes/week</td>
<td>132 tonnes/week</td>
<td>3</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td></td>
<td>( Z_{414, \text{opt}} )</td>
<td>189 tonnes/week</td>
<td>132 tonnes/week</td>
<td>4</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>Recyling</td>
<td>( Z_{311, \text{opt}} )</td>
<td>350 tonnes/week</td>
<td>140 tonnes/week</td>
<td>1</td>
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<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td></td>
<td>( Z_{313, \text{opt}} )</td>
<td>350 tonnes/week</td>
<td>140 tonnes/week</td>
<td>3</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td></td>
<td>( Z_{314, \text{opt}} )</td>
<td>350 tonnes/week</td>
<td>140 tonnes/week</td>
<td>4</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td></td>
<td>( Z_{315, \text{opt}} )</td>
<td>350 tonnes/week</td>
<td>140 tonnes/week</td>
<td>5</td>
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<td>0</td>
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<td>[1, 1]</td>
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<td>Expansion cost (( \text{$10}^6 ))</td>
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<td>12.44, 14.08</td>
<td>11.70, 13.19</td>
<td>11.56, 13.02</td>
<td>11.38, 12.82</td>
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Table 6 – Solution of the ITOM for continuous variables

<table>
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<tr>
<th>Period</th>
<th>Facility</th>
<th>Level of waste generation (%)</th>
<th>Probability (%)</th>
<th>Allowable waste flow (tonnes/week)</th>
<th>q=0.01 (tonnes/week)</th>
<th>q=0.05 (tonnes/week)</th>
<th>q=0.1 (tonnes/week)</th>
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<tbody>
<tr>
<td></td>
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<td>Excess waste flow</td>
<td>Optimized waste flow</td>
<td>Excess waste flow</td>
<td>Optimized waste flow</td>
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<td>k=1</td>
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(continued on next page)
Table 6 (continued)

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<thead>
<tr>
<th>Period</th>
<th>Facility (i)</th>
<th>Level of waste generation</th>
<th>Probability (%)</th>
<th>Allowable waste flow (tonnes/week)</th>
<th>q=0.01 (tonnes/week)</th>
<th>q=0.05 (tonnes/week)</th>
<th>q=0.1 (tonnes/week)</th>
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<tr>
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<td>Excess waste flow</td>
<td>Optimized waste flow</td>
<td>Excess waste flow</td>
</tr>
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<td>[580, 592]</td>
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Expected system cost ($):
with \( q_i \) level. The diversion rates (i.e. waste flows to the composting and recycling facilities) would be \([47.80, 48.25]\)% when \( q_i = 0.01 \), \([47.28, 47.70]\)% when \( q_i = 0.05 \), and \([47.04, 47.47]\)% when \( q_i = 0.10 \). The diversion rate would keep decreasing when \( q_i \) level is increased.

The remaining landfill capacities at the end of the planning horizon would be \([837.95, 938.16]\)×10³, \([826.20, 929.23]\)×10³ and \([819.46, 925.26]\)×10³ tonnes when \( q_i = 0.01 \), \( q_i = 0.05 \) and \( q_i = 0.10 \), respectively, demonstrating a decreasing tendency along with an increasing \( q_i \) level. This is due to the following facts: (i) an increased \( q_i \) level means a relaxed diversion-rate constraint and thus a raised risk in violating the diversion requirements, leading to more excess wastes to the landfill; (ii) since the landfill has lower regular and penalty costs than the composting and recycling facilities, it would often be the desired target for accepting the excess wastes. The results also indicate that only a small portion of excess wastes would be diverted to the recycling facility since this facility has a high penalty for excess flows.

Variations in the \( q_i \) levels also correspond to the decision makers’ preferences regarding the tradeoff between the system-cost and the constraint–violation risk. The resulting system costs (\( f^\pm \)) are \([105.23, 131.29]\)×10⁶, \([104.58, 130.48]\)×10⁶ and \([104.22, 130.04]\)×10⁶ under \( q_i = 0.01 \), \( q_i = 0.05 \) and \( q_i = 0.10 \), respectively (Table 6). As the actual values of the variables and/or parameters vary within their two bounds, the expected system cost would change correspondingly between \( f_{\text{opt}} \) and \( f_{\text{opt}}^+ \) with different reliability levels. Fig. 3 shows the trend of system-cost variations under different \( q_i \) levels. It is indicated that a lower \( q_i \) level corresponds to a higher system reliability and, meanwhile, leads to a higher system cost; in comparison, a higher \( q_i \) level would result in a lower cost but a higher constraint-violation risk.

The system cost includes expenses for expansions/developments of landfilling, composting and recycling facilities, disposal of wastes at the landfill, and diversion wastes to the composting and recycling facilities. Fig. 4 presents the detailed costs for waste management under different significance levels. The costs for facility expansions would be \([11.70, 13.19]\)×10⁶ (or \([10.0, 11.1]\) of the total system cost) when \( q_i = 0.01 \), \([11.56, 13.02]\)×10⁶ (or \([10.0, 11.1]\) of the total) when \( q_i = 0.05 \), and \([11.38, 12.82]\)×10⁶ (or \([9.9, 10.9]\) of the total) when \( q_i = 0.10 \); this demonstrates that a raised \( q_i \) level would lead to a reduced expansion cost. The costs for landfilling wastes are \([36.38, 49.73]\)×10⁶ (i.e. \([34.6, 37.9]\) of the total) when \( q_i = 0.01 \), \([36.8, 50.51]\)×10⁶ (i.e. \([35.2, 39.0]\) of the total) when \( q_i = 0.05 \), and \([36.96, 50.79]\)×10⁶ (i.e. \([35.5, 39.0]\) of the total) when \( q_i = 0.10 \); this indicates that a raised \( q_i \) level would result an increased operating cost at the landfill. The diversion costs for wastes treated at both composting and recycling facilities are \([57.15, 68.37]\)×10⁶ (or \([52.1, 54.3]\) of the total) when \( q_i = 0.01 \), \([56.22, 66.95]\)×10⁶ (or \([51.3, 53.8]\) of the total) when \( q_i = 0.05 \), and \([55.88, 66.43]\)×10⁶ (or \([51.1, 53.6]\) of the total) when \( q_i = 0.10 \); the corresponding diversion rates would be \( 48.1\% \) when \( q_i = 0.01 \), \( 47.5\% \) when \( q_i = 0.05 \), and \( 47.2\% \) when \( q_i = 0.10 \), implying that a higher \( q_i \) level would sacrifice the environmental objective but correspond to reduced diversion costs. Therefore, decisions with a lower \( q_i \) level would possess a higher system reliability but a higher system cost as well; conversely, a desire for reducing the system cost would result in an increased risk of violating the constraints.
On the other hand, the operating costs include expenses for handling fixed allowable and probabilistic excess waste flows. The regular cost for the disposal of allowable-waste flows would be $88.36, 108.98 \times 10^6$ under all significance levels. The penalty costs for handling excess flows would be $5.17, 9.12 \times 10^6 (q_i=0.01)$, $[4.66, 8.48] \times 10^6 (q_i=0.05)$ and $[4.48, 8.24] \times 10^6 (q_i=0.10)$, demonstrating a decreasing tendency along with the increasing $q_i$ level. This is because more excess flows would be shipped to the landfill when the $q_i$ level is raised (i.e. relaxed diversion constraints), while the landfill has lower penalty costs for handling the excess flows than the composting and recycling facilities.

The developed model could provide a linkage to the pre-regulated policies that have to be respected when a modeling effort is undertaken. The complexity associated with the allowable-waste-flow levels is mainly caused by the limited capacity for waste disposal and the increasing waste-generation amount. Therefore, variations in the values of $X_{ik}$ (allowable-waste flow to facility $i$ during planning period $k$) would lead to multiple scenarios corresponding to different policies for managing waste generation, reduction and recycling, as well as waste diversion and disposal. For example, if the allowable-waste flows are regulated at too low levels, then high penalties may have to be paid when the allowances are violated; conversely, if the allowable-waste-flow levels are too high, then the planned capacities for waste management would become too large, leading to waste of capital for capacity expansion and/or development. From a long-term planning point of view, municipal waste-generation rates would keep rising due to population increase and economic development, and the available capacities of waste-management-facilities may also vary among different time periods. Thus, another complexity is associated with the dynamic variations of system capacity due to the expansions and/or developments of waste-management-facilities.

5. Conclusions

A solid waste decision-support system based on an integrated two-stage optimization model (ITOM) has been developed for the City of Regina, Canada. The ITOM incorporates the approaches of two-stage stochastic programming, chance-constrained programming, and interval mathematical programming within an integer programming framework. It can reflect the dynamic, interactive and uncertain characteristics of the solid waste-management system in the study city, and can address issues concerning waste diversion and landfill prolongation as deemed critical by the local authority.

Violations for capacity and diversion constraints are allowed under a range of significance levels, which reflect the tradeoffs between system-cost and constraint-violation risk. Reasonable solutions for both binary and continuous variables have been generated. They represent decisions of facility expansion and waste-flow allocation. These inexact solutions can be used to generate multiple decision alternatives, such that desired policies under various environmental, economic, and system-reliability conditions can eventually be identified. Decisions at a lower significance level would lead to a higher system reliability and, at the same time, a higher system cost; conversely, a desire for reducing the system cost would result in an increased risk of violating the constraints.

Owing to the complex nature of the waste-management system, the data required for defining different scenarios were extensive. Although most of the obtained data are relatively accurate, others are less so. Therefore, increasing the certainty of the data sets through further investigation and verification would help increase the certainty of the generated solutions. Moreover, extension of the ITOM to a wide scope of waste-management problem for both residential and non-residential sectors would be an interesting topic that deserves future research efforts.

Acknowledgments

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Appendix A. Modeling formulation for MSW management

Based on the city’s waste-management policies, an allowable-waste-flow level from the city to each facility is pre-regulated. If this level is not exceeded, it will result in a regular cost to the system. If it is exceeded, however, the surplus waste has to be handled through more expensive manners, resulting in an excess cost (or penalty) to the system (i.e., excess flow = generated flow – assigned quota). The excess cost may be represented in terms of raised collection, transportation and/or operation costs. The study time horizon is 25 years, consisting of five 5-year periods. Consequently, the objective for the entire system is to minimize the net total cost for facility expansion/development and waste disposal over a long-term planning horizon, subject to various constraints related to waste generation, disposal capacity, and diversion policy. Thus, we have:

\[ \min \quad f^* = \sum_{i=1}^{5} \sum_{k=1}^{T_i} L_i (T_{ik}^f + \text{OP}_{ik}^f) + \sum_{i=1}^{5} \sum_{k=1}^{T_i} L_i (T_{ik}^f E_k^f F_{ik}^f T_{ik}^f) + \sum_{i=1}^{5} \sum_{k=1}^{T_i} L_i (T_{ik}^f E_k^f D_{ik}^f F_{ik}^f T_{ik}^f)
\]

subject to:

\[ \sum_{k=1}^{T_i} \left[ \sum_{i=1}^{5} (T_{ik}^f + X_{ik}^f) \right] \leq (\text{FE}_{ik}^f)^{\text{max}} \] \[ \forall k \text{, } k' \text{, } h \text{, } \forall i = 1, 2, 3, 4, 5; \] \[ \text{[Total landfill capacity constraints]} \] \[ \sum_{k=1}^{T_i} (T_{ik}^f + X_{ik}^f) \leq (\text{FC}_{ik}^f)^{\text{max}} X_{ik}^f \] \[ \forall k \text{, } k' \text{, } h \text{, } \forall i = 1, 2, 3, 4, 5; \] \[ \text{[Capacity constraints of the northern-site landfill]} \] \[ T_{ik}^d + X_{ik}^d \leq (\text{TC}_{ik}^d)^{\text{max}} \] \[ \forall k' \text{, } h \text{, } \forall i = 1, 2, 3, 4, 5; \] \[ \text{[Composting capacity constraints]} \] \[ (T_{ik}^f + X_{ik}^f) \leq (\text{TC}_{ik}^f)^{\text{max}} \] \[ \forall k' \text{, } h \text{, } \forall i = 1, 2, 3, 4, 5; \] \[ \text{[Recycling capacity constraints]} \] \[ \sum_{i=1}^{5} (T_{ik}^f + X_{ik}^f) \leq (\text{DG}_{ik}^d)^{\text{max}} \] \[ \forall k \text{, } h \text{, } \forall i = 1, 2, 3 \] \[ \text{[Waste disposal demand constraints]} \] \[ 0 \leq X_{ik}^d \leq T_{ik}^d \] \[ \forall k \text{, } h \text{, } \forall i = 1, 2, 3 \] \[ \text{[Non-negativity and technical constraints]} \] \[ (\text{DG}_{ik}^d)^{\text{max}} \] \[ \text{[Binary constraints]} \] \[ \sum_{i=1}^{5} Y_{ik} \leq 1 \] \[ \text{[Landfill expansion/development may only be considered once in the planning horizon]} \] \[ \sum_{i=1}^{5} Z_{ik} \leq 1 \text{, } i = 4, 5 \] \[ \text{[Expansions for composting and recycling facilities may occur in any given time period]} \]

where \( f^* \) is expected system cost ($); \( L_i \) is length of time period \( k \) (week); \( k \) is time period, \( k = 1, 2, 3, 4, 5 \); \( m \) is name of expansion option for composting and recycling facilities, \( m = 1, 2, 3, 4, 5 \); \( h \) is level of waste generation, \( h = 1, 2, 3, 4, 5 \) when the waste-generation rate is low, low-medium, medium, medium-high and high, respectively; \( i \) is type of waste-management-facility, \( i = 1, 2, 3, 4, 5 \); \( D_{ik}^f \) is cost of collecting and transporting excess waste from the city to facility \( i \) during period \( k \) ($/tonne) (the second-stage cost parameter), where \( D_{ik}^f \geq D_{ik}^d \); \( F_{ik}^f \) is cost of collecting and transporting excess waste from the landfill, \( i = 1, 2, 3, 4, 5 \) during period \( k \) ($/tonne) (the second-stage cost parameter), where \( F_{ik}^f = F_{ik}^d \); \( T_{ik}^d \) is disposal cost for excess waste residues generated by the composting or recycling facility during period \( k \) ($/tonne) (the second-stage cost parameter), where \( T_{ik}^d \leq T_{ik}^d \); \( F_{ik}^f \) is residue flow from facility \( i \) to the landfill ($/tonne); \( m \) is expanded/developed capacity of the landfill corresponding to significant level \( q_i \) ($/tonne); \( (\text{DG}_{ik}^d)^{\text{max}} \) is expanded/developed capacity of the landfill corresponding to significant level \( q_i \) ($/tonne); \( (\text{TC}_{ik}^f)^{\text{max}} \) is expanded/developed capacity of the landfill corresponding to significant level \( q_i \) ($/tonne).
to significance level \( q_i \) (tonne), \( i = 1, 2, 3 \); \( OP_k \) is regular operating cost of waste-management-facility \( i \) during period \( k \) ($/tonne), \( i = 1, 2, 3, 4, 5 \); \( p_k \) is probability of waste-generation rate \( \omega_{ih} \) with level \( h \) (%); \( q_i \) is probability of violating constraint \( i \) (%); \( RE_k \) is revenue generated by processing allowable-waste flows in composting or recycling facility during period \( k \) ($/tonne), \( i = 4, 5 \); \( RM_k \) is revenue generated by processing excess waste flows in composting or recycling facility during period \( k \) ($/tonne), \( i = 1, 2, 3, 4, 5 \); \( TR_k \) is cost of collection and transportation for allowable-waste flow to facility \( i \) during period \( k \) (tonne/week), \( i = 4, 5 \); \( TC_k \) is maximum allowable-waste flow from the city to facility \( i \) during period \( k \) (tonne/week); \( TC_i \) is existing capacity of composting or recycling facility corresponding to significance level \( q_i \) (tonne/week), \( i = 4, 5 \); \( \Delta TC_i \) is expanded/developed capacity for composting or recycling facility with expansion option \( m \) in period \( k \) corresponding to significance level \( q_i \) (tonne/week), \( i = 4, 5 \); \( TR_k \) is cost of collection and transportation for allowable-waste flow to facility \( i \) during period \( k \) (tonne/week), \( i = 1, 2, 3, 4, 5 \); \( \omega_{ih} \) is residential waste-generation rate with probability \( p_k \) in period \( k \) (tonne/week); \( X_{kh} \) is amount by which the allowable-waste level is exceeded when the waste-generation rate is \( \omega_{kh} \) with probability \( p_k \) (tonne/week); \( Y_{kh} \) is binary decision variable for landfill expansion/development at the start of period \( k \), \( i = 1, 2, 3 \); \( Z_{km} \) is binary decision variable for expanding composting or recycling facility with option \( m \) at the start of period \( k \), \( i = 4, 5 \).

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