Solid waste management in urban areas
Development and application of a decision support system

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Abstract

A decision support system (DSS) developed to assist the planner in decisions concerning the overall management of solid waste at a municipal scale is described. The DSS allows to plan the optimal number of landfills and treatment plants, and to determine the optimal quantities and the characteristics of the refuse that has to be sent to treatment plants, to landfills and to recycling. The application of the DSS is based on the solution of a constrained non-linear optimization problem. Various classes of constraints have been introduced in the problem formulation, taking into account the regulations about the minimum requirements for recycling, incineration process requirements, sanitary landfill conservation, and mass balance. The cost function to be minimized includes recycling, transportation and maintenance costs. The DSS has been tested on the municipality of Genova, Italy, and the results obtained are presented.

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1. Introduction

Nowadays, solid waste management is a problem of major relevance for all societies. Moreover, finding acceptable strategies to cope with such a problem is becoming a quite hard task, owing to the increasing awareness of environmental issues by population and authorities. In general, such a consciousness has led to the development of pollution control technologies and to a more rigorous legislation on waste handling and disposal, to minimize the environmental impact associated with solid waste. In particular, European legislation recommends the development of local integrated management plans, which give priority to prevention, waste reduction and recovery, and allow using landfill only for the disposal of refuses that cannot be recovered. In Italian municipalities, the new legislation, the rapid increase of solid waste production and the frequent landfill closings have encouraged the development of incineration and recycling programs. The definition of such programs must take into account an integration of economic, environmental, social and technical considerations.

The aim of this work is that of presenting the structure and the application of a decision support system (DSS) designed to help decision makers of a municipality in the development of incineration, disposal, treatment and recycling integrated programs. The main alternatives to treat collected municipal solid waste (MSW) are represented by recycling, treatment in specific plants, and sanitary landfill disposal. Specifically, within a MSW management system, several treatment plants and facilities can generally be found: separators, plants for production of refuse derived fuel (RDF), incinerators with energy recovery, plants for treatment of organic material, and sanitary landfills. One of the specific objectives of this work is to optimize the MSW flows, thus defining the quantities (per unit time) and the characteristics of refuse that has to be sent to recycling, to the different treatment plants or to the sanitary landfills.

In Section 2, a brief survey is presented of the main approaches proposed in the literature for solid waste management in urban areas. Section 3 presents a general description of the physical model on which the DSS is based, while in Section 4, the mathematical model and the optimization problem are presented in detail, specifying the cost function, the decision variables and the problem constraints. A brief description of the case study of the municipality of Genova (Italy), and the results of the application of the proposed procedure, are presented in Section 5.

2. Optimization models for MSW management

MSW management is a complex, multidisciplinary problem involving economic and technical aspects, normative constraints about the minimum requirements for recycling and sustainable development issues. Most industrialized countries have adopted the philosophy of the ‘Waste Management Hierarchy’ (prevention/minimization, materials recovery, incineration and landfill) as a guide for developing MSW management strategies (Sakai et al., 1996). To formalize these strategies, in the
last two decades, considerable research efforts have been directed towards the
development of economic-based optimization models for MSW flow allocation.
Several examples of mathematical programming models have been developed for
MSW management planning. A model based on the minimization of overall cost,
taking into account energy and material recovery requirements, formulated as a
constrained non-linear optimization problem, has been recently presented (Chang
and Chang, 1998). In this model, the cost function includes transportation,
treatment, maintenance and recycling costs, and possible benefits for electric energy
sales. Decision variables are continuous and represent the material flows to the
various plants. The number and the typologies of such plants are defined a priori; in
other words, the model does not provide automatically the optimal structure of the
MSW management system. The problem constraints are relevant to mass balance,
plant capacities, and the required minimum energy recovery. However, the model
does not take into account normative, environmental and technical aspects, and it
does not include organic material recovery and production of RDF sold to power
plants.

Another approach (Daskalopoulos et al., 1998) has been formulated for the
management of MSW flows, taking into account their rates and composition, and
their adverse environmental impacts. In this model, the optimal MSW flows to
desert typologies of treatment plants (landfill, incineration with energy recovery,
waste recycling and waste composting) are determined by minimizing a linear cost
function, assuming that flows are independent variables.

In Italy (D’Antonio and Fabbricino, 1998), a multiple choice approach has been
proposed and applied. Such an approach integrates the costs of treatment plant
localization with the costs of transport and treatment of MSW. The minimum cost
solution is determined using a mathematical model whose formalization includes
constraints deriving from technical reasons and legislation. Although all flows and
costs are accurately defined, the MSW heating value, which represents an important
aspect for the incineration process, is not taken into account. In addition, other
important technical issues related to MSW management system, such as the
production of RDF and the evaluation of the time for landfill saturation, are not
included in the model.

Generally speaking, in planning a MSW management system, a fundamental
difficulty is that of taking simultaneously into account conflicting objectives (which
in general cannot be dealt with by economical quantifications only); in addition, the
problem is characterized by an intrinsic uncertainty of the estimates of costs and
environmental impacts. Such reasons have led several authors (Hokkanen and
Salminem, 1997; Karagiannidis and Moussiopoulos, 1997; Chang and Wang,
1997a,b) to propose multi-criteria decision approaches, sometimes allowing the
formal representation of uncertain or imprecise information.

In this connection, the development of computer-integrated tools, capable of
assisting the planner in the various steps of the design procedure, is becoming
increasingly interesting. For instance, Haastrup et al. (1998) have recently developed
and applied a DSS integrated with a MSW information system. Such a system
includes three main interacting components: the user interface system, the data
management system (geographical data, data related to waste treatment, disposal and waste production), and the model management system. Specifically, the DSS includes: a vulnerability map concerning waste treatment sites, four models (the site risk, the environmental impact, the costs, and the transportation risk models) for scenario evaluations, and the NAIADE (Novel Approach to Imprecise Assessment and Decision Environments) model for multicriteria analysis. A further example of a computer model for MSW integrated management can be found in Wang et al. (1996).

It is obviously quite difficult to focus on all aspects of a waste management problem. A current research trend is both to focus deeply on some specific aspects and to experiment specific optimization techniques to solve the problem. For example, a model addressing detailed economic considerations (Masui et al., 2000) has been recently developed. In that work, the input–output analyses are expanded to dynamic optimization, since the process of the disposal shortage is modeled as a dynamic process. An example of research based on different DSS techniques is the approach based on a genetic algorithm allowing the generation of multiple network configurations (that are policy alternatives) to be evaluated by human decision-makers (Rubenstein-Montano et al., 2000). Finally, it is interesting to note that problems similar to those described in this paper, but at a national scale, can be studied by use of the same approach developed for the municipal scale (Ljunggren, 2000).

In this framework, the purpose of the present paper is that of considering a general model, which is comprehensive of all technical and economical aspects concerning the management of MSW. The various possible kinds of plants and processes for treatment of solid waste are considered. A specific feature of the proposed model is the presence of a set of binary decision variables whose value indicates the presence/absence of some specific plants in the final configuration of the MSW management system. Thus, formalizing and solving the mathematical programming problem that will be introduced in Section 4 allows to define the optimal configuration of the system (from the physical and technological point of view), as well as to determine the optimal flow among the various plants in the system.

3. The physical model of the MSW management system

To build a complete model of MSW management systems, a wide knowledge and a deep analysis of the possible treatment processes of the materials composing the refuse are needed. For example, the total amount of treated waste can have different composition, according to different plans for solid waste treatment and to the availability of specific treatment plants. Since waste that is not recycled should be treated, sizing of recycling processes is strictly related to the availability of treatment plants. A trade-off analysis about the convenience to increase recycling versus the need to build new treatment plants is not easy, as it depends on many factors, such as the types and possible locations of treatment plants, economic considerations,
environmental preservation, etc. Then, to take a decision taking into account all such issues, it is necessary to accurately model the system, analyzing recovery versus disposal of every material, and representing the MSW flows, as well as their chemical and physical characteristics. In fact, the modeling of such characteristics (as humidity and heating value) is needed in order to evaluate the possible energy and material recovery from them. Note that energy and material recoveries may be conflicting objectives: for example, if materials with high heating values are completely recycled, the remaining refuse is likely to be not very significant from an energetic point of view.

In the proposed model, the MSW is supposed to be composed by eight kinds of materials (paper, plastic, glass, organic material, metals, wood, inert matter and scraps). The waste composition and/or the waste heating value before and after any treatment plant is represented through specific quantities that are explicitly considered in the model. The refuse flows, for each kind of material, which enter and go out from each plant depend on the decision variables whose values have to be determined in the design of the MSW management system.

Fig. 1 provides a basic scheme of the flows in a MSW management system. The daily production $R$ can be recovered by recycling or it can be sent to three kinds of treatment plants: separators, incinerators and landfills. Three flows come out from a separator:

- the metals that can be sent to recycling;
- the organic material sent to treatment plants;
- a fraction of material, with low humidity and high heating value, that can be burnt, or sent to plants for RDF production, or disposed in a sanitary landfill.

Finally, the RDF plant produces RDF, which can be sold to industries, and scraps, which can be sent either to an incinerator or to a landfill. In this model, recycling is possible for six kinds of materials: paper, plastic, glass, wood, metals and organic material. These materials can be separately collected by different methods. The organic material collected for recycling can be directly sent to a composting plant because it is pure enough to produce compost for agricultural use. Material recovery can also be obtained in plants for MSW treatment. To reach this goal, three possible typologies of treatment plants are considered: separators, plants for RDF production, and plants for organic material treatment. Specifically, a separator can mechanically extract organic materials from the materials that can be used for RDF production or for energy recovery by incineration, and, by specific machinery, it can also efficiently recover metals. A plant for RDF production mechanically selects materials with high heating value, such as paper and plastic, and the result is a combustible that can be sold. Finally, a plant for organic material treatment treats the organic material coming from a separator, so that it can be recovered and re-used to cover full landfills or for the reclamation of a piece of land and scraps.

In this model also energy recovery by MSW combustion is represented. Such an issue gives even more difficulties in MSW management. Actually, recycling affects the heating value of the refuse that should be burnt because it influences the
composition of the refuse sent to incineration. In fact, the withdrawal of different kinds of materials, and a high production level of RDF (composed in particular by plastic and paper) decrease the quantities of plastic and paper that are sent to incinerators. On the other hand, energy production by an incinerator heavily depends on the composition and on the heating value of the refuse entering the incinerator plants.

Finally, the model includes also the possibility of disposal in sanitary landfills. In this model, such a disposal may be constrained by a maximum amount of MSW flow that can be sent to the landfill, or equivalently by a minimum number of years for the complete filling of the landfill.

Fig. 1. General representation of the model.
4. The mathematical optimization model

The main DSS goal is to plan the MSW management, defining the refuse flows that have to be sent to recycling or to the different treatment or disposal plants, and suggesting the optimal number and the kinds of plants that have to be active. To this end, it is necessary to define the decision variables of the problem, the cost function to be minimized, and the various constraints affecting the problem.

4.1. Detailed description of the model

Seven typologies of materials have been taken into account (0, inert matter and scraps; 1, paper; 2, plastic; 3, glass; 4, organic; 5, wood; and 6, metals). The total daily MSW production is $R$; $r_i$ is the daily quantity of material of type $i$. All types of materials $i = 1, \ldots, 6$ are recyclable. A detailed representation of the model is depicted in Fig. 2. Recyclable materials present in the produced refuse can be recycled after separate collection. The material flows to be sent to recycling represent a first set of decision variables. For the sake of clarity, in writing the overall objective function and the whole set of constraints, all decision variables are highlighted in bold style. More specifically, let us introduce the variable $\zeta_i$, which represents the percentage of material of type $i$ sent to recycling ($i = 1, \ldots, 6$). For the sake of notational completeness, let $\zeta_0 = 0$. The remaining refuse flow (which is not recycled) is collected and taken to the generic $P_d$ ($d = 1, \ldots, D$) collecting site. Let $\chi_{P_d,i}$ be the fraction of $r_i$ which is collected at collection site $P_d$. The number and location of
such sites are fixed a priori, as they are assumed to be already existing. For plants of other kinds (to be detailed below), their possible locations are assumed to have been fixed, as well as their physical characteristics, sizes, etc. Thus, the only issue to be specified remains their existence or not, which will be modeled through binary decision variables.

From a collecting site, the refuse can be sent either to a separator \( S_p \) (\( p = 1, \ldots, P \)), or to an incinerator \( I_n \) (\( n = 1, \ldots, N \)), or to a landfill \( L_m \) (\( m = 1, \ldots, M \)). Then, it is necessary to introduce the decision variables \( \varphi_{P_d,L_m} \), \( \varphi_{P_d,S_p} \), \( \varphi_{P_d,I_n} \), which correspond to the percentages of the untreated waste coming from collecting site \( P_d \) and sent, respectively, to landfill \( L_m \), to separator \( S_p \), and to incinerator \( I_n \). Clearly, such percentages are the same for all kinds of materials, and thus they not depend on index \( i \).

A separator divides the refuse in three parts: metals sent to recycling, humid material sent to specific organic material treatment plants \( T_p \), \( p = 1, \ldots, P \) (it is supposed that for any separator there is a corresponding organic material plant), and a remaining quantity of dry waste, which can be sent to RDF-plants, \( (C_q, q = 1, \ldots, Q) \), or to incinerators (\( I_n, n = 1, \ldots, N \)), or to landfills (\( L_m, m = 1, \ldots, M \)). The decision variables \( \psi_{S_p,C_q}, \psi_{S_p,I_n}, \psi_{S_p,L_m} \) correspond to the fractions of dry material coming from separator \( S_p \) (\( p = 1, \ldots, P \)) and sent, respectively, to RDF-plant \( C_q \), to incinerator \( I_n \), and to landfill \( L_m \).

A RDF-plant separates entering refuse into two parts: RDF that can be sold, and scraps that can be sent to incinerators (\( I_n \)) and to landfills (\( L_m \)). Variables \( \lambda_{C_q,L_m} \) and \( \lambda_{I_n,L_m} \) represent fractions of scraps coming from RDF-plant \( C_q \) and sent, respectively, to landfill \( L_m \) and to incinerator \( I_n \).

Ashes produced by incinerators \( I_n \) are sent to landfills; let \( \sigma_{I_n,L_m} \) be the fractions of ashes coming from incinerator \( I_n \) and sent to landfill \( L_m \).

Scraps coming from organic material plants \( T_p \) can be sent to the different landfills positioned on the territory. Variable \( \gamma_{T_p,L_m} \) represents the fraction of scraps coming from organic material plant \( T_p \) and sent to landfill \( L_m \). Binary decision variables are associated to all separators, RDF plants, incinerators and landfills to indicate, through their value, the presence/absence of such plants: \( \delta_{S_p}, \delta_{C_q}, \delta_{I_n}, \delta_{L_m} \).

The list of decision variables is reported in Appendix A. Note that binary as well as continuous variables are needed in this model.

### 4.2. Constraints

Six classes of constraints have been included in the formalization of the mathematical optimization problem. The first class of constraints derives from normative issues about the minimum requirements for recycling (see Section 4.2.1). The second one derives from technical aspects of the incineration process (see Section 4.2.2). The third one derives from the necessity of avoiding a too rapid fulfilling of sanitary landfills (see Section 4.2.3). The fourth class of constraints simply derives from flow conservation (see Section 4.2.4). The fifth class of constraints has only a logical meaning, as it imposes the existence of a plant when...
4.2.1. Normative constraints

Italian legislation requires that waste recycling is no less than 35% of the total produced waste in mass. Note that this constraint is necessary since, as it will be pointed out in Section 4.3, recycling is not advantageous from the economic point of view. Materials contributing to reach this percentage can be recovered either by recycling (given by separately collected materials, which are directly sent to recycling and by metals recovered by the separator) or by treatment plants (constraint (1)). Thus, in our model, it is necessary to impose that:

\[(\text{materials recovered by separate collection}) + (\text{materials present in the MSW sent to the separators and recovered with efficiency } k_i) + (\text{stabilized organic material}) + (\text{produced RDF}) \geq 35\% \text{ of the total refuse}\]

\[
\sum_{i=1}^{6} \alpha_i r_i + \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{i=1}^{6} k_i (1 - \alpha_i) r_i \varphi_{p_d} \varphi_{p_s} + \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{i=0}^{6} (1 - \alpha_i) r_i \varphi_{p_d} \varphi_{p_s} (1 - k_i) \eta_{S_d} \tilde{\eta}_{T_d} \sum_{i=0}^{6} r_i, \quad (1)
\]

where \(k_i\) is the fraction of material \(i\) sent to recycling after separation (it is assumed that this fraction is the same for all separators); \(\eta_{S_d,i}\), fraction of material \(i\), with respect to the overall material not recycled after separation in \(S_p\), which is sent (as humid material) to the organic treatment plant \(T_p\); \(\tilde{\eta}_{T_d,i}\), parameter that represents the fraction of material \(i\), entering organic material plant \(T_p\), which remains included in the stabilized organic material; note that such a fraction depends on index \(i\) because clearing operations to eliminate scraps in organic material depends on index \(i\); \(\tilde{\eta}_{C_q,i}\), fraction of material \(i\), entering the RDF plant \(C_q\), which remains as a constituent of RDF produced at that plant; here also, there is a dependence on index \(i\) because clearing operations to eliminate scraps depends on index \(i\).

Another normative constraint is relevant to the RDF heating value that has to reach a minimum value \(c\) of 3600 kcal/kg. The proposed simplified formulation to compute RDF heating value depends on heating value and humidity of every material composing the refuse: it is evaluated as a weighted average of the contributions of the various materials, obtained taking into account the heat needed to evaporate the water present in the refuse. Applying this formulation, it is possible to formalize one constraint for each RDF plant as follows:

the heating value per kg of the RDF produced by the RDF plant \(C_q\) must be greater than or equal to a fixed lower bound
\[
\frac{1}{\text{RDF}_{C_q}} \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{i=0}^{6} \left( (1 - \alpha_i) r_i \chi_{P_{d,i}} \psi_{P_{d,i}} S_p (1 - k_i) (1 - \eta_{S_{p,i}}) \psi_{S_{p,i}} C_q \hat{\lambda}_{C_q,i} \right) \]

\[
[(\text{HV}_i (1 - \text{HUM}_i) - G \cdot \text{HUM}_i)] \geq c,
\]

(2)

where \( \text{HUM}_i \) is the humidity, i.e. mass fraction of water contained in the \( i \)th material; \( \text{HV}_i \), heating value (kcal/kg) for the \( i \)th material, considering the \( i \)th material as dry; \( G \), heat necessary to evaporate 1 kg of water (606.5 kcal/kg);

\( \text{RDF}_{C_q} \), daily quantity of produced RDF in \( C_q \), given by the following formulation:

\[
\text{RDF}_{C_q} = \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{i=0}^{6} \left( (1 - \alpha_i) r_i \chi_{P_{d,i}} \psi_{P_{d,i}} S_p (1 - k_i) (1 - \eta_{S_{p,i}}) \psi_{S_{p,i}} C_q \hat{\lambda}_{C_q,i} \right).
\]

(2a)

4.2.2. Technical constraints for incinerator plants

The incinerators can receive only waste satisfying some requirements. Specifically, an incinerator can efficiently work if minimum and maximum mass (constraint (3)) and specific heating value (constraints (4–6)) are guaranteed. The mass constraint specifies that, for each incinerator:

\( \text{hourly refuse directly sent to an incinerator } I_n \) (coming from collection sites, separators, or RDF plants) must fall within specified lower and upper bounds (respectively, \( M_{I_n,a} \) and \( M_{I_n,b} \)):

\[
M_{I_n,a} \delta_{I_n} \leq \frac{1}{24} \left( \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{i=0}^{6} (1 - \alpha_i) r_i \chi_{P_{d,i}} \psi_{P_{d,i}} I_n \right) + \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{i=0}^{6} (1 - \alpha_i) r_i \chi_{P_{d,i}} \psi_{P_{d,i}} S_p \psi_{S_p} I_n (1 - k_i) (1 - \eta_{S_{p,i}}) + \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{i=0}^{6} (1 - \alpha_i) r_i \chi_{P_{d,i}} \psi_{P_{d,i}} S_p \psi_{S_p} C_q (1 - \hat{\lambda}_{C_q,i}) \right) \leq M_{I_n,b} \delta_{I_n}.
\]

(3)

Note that constraint (3) allows a non-zero refuse flow to incinerator \( I_n \) only when such a plant is actually existing (i.e. \( \delta_{I_n} = 1 \). The heating value (HV) of the overall material entering an incinerator must also fall within two fixed bounds, namely \( a_{I_n} \) and \( b_{I_n} \), expressed in kcal/kg. In our model, we impose that such a constraint must be separately verified by all waste flows that enter the incinerator \( I_n \). This (actually quite restrictive) point of view is justified by the necessity of continuously feeding the incinerator with materials whose flow may suffer from some discontinuity. For this reason, it is necessary to impose the above constraint over all kinds of flows entering
For this reason, it is necessary to introduce in our model specific constraints, which for the scraps coming from RDF
plants.

\[
\frac{D}{d=1} \sum_{i=0}^{6} (1 - \alpha_i) \varphi'_{P_d,i} \varphi_{P_d,i} \Delta t (HV_i(1 - HUM_i) - G \cdot HUM_i) 
\leq b_i \delta_t \tag{4}
\]

for the HV of untreated MSW

\[
\frac{D}{d=1} \sum_{p=1}^{P} \sum_{i=0}^{6} (1 - \alpha_i) \varphi'_{P_d,i} \varphi_{P_d,i} (1 - k_i)(1 - \eta_{S_p,i}(HV_i(1 - HUM_i) - G \cdot HUM_i) 
\leq b_i \delta_t \tag{5}
\]

for the HV of refuse coming from the separators (the fraction without organic material)

\[
\frac{D}{d=1} \sum_{p=1}^{P} \sum_{n=1}^{N} \sum_{i=0}^{6} (1 - \alpha_i) \varphi'_{P_d,i} \varphi_{P_d,i} (1 - k_i)(1 - \eta_{S_p,i}) \varphi_{S_p,i} (1 - \eta_{C_q,i}) \varphi_{C_q,i} (HV_i(1 - HUM_i) - G \cdot HUM_i) 
\leq b_i \delta_t \tag{6}
\]

for the scraps coming from RDF-plants.

4.2.3. Constraints related to the material flows sent to sanitary landfills

Solutions for MSW management problems that are heavily based on sanitary landfill exploitation are not environmentally sustainable over a long time horizon. For this reason, it is necessary to introduce in our model specific constraints, which have the function of preventing a too rapid saturation of the available sanitary landfills. Such constraints may be expressed in terms of the minimum filling time for each sanitary landfill.

\[
\sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i) \varphi'_{P_d,i} \varphi_{P_d,i} \Delta t (HV_i(1 - HUM_i) - G \cdot HUM_i) 
\leq b_i \delta_t
\]

for the HV of untreated MSW

\[
\sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{i=0}^{6} (1 - \alpha_i) \varphi'_{P_d,i} \varphi_{P_d,i} (1 - k_i)(1 - \eta_{S_p,i}) \varphi_{S_p,i} (1 - \eta_{C_q,i}) \varphi_{C_q,i} (HV_i(1 - HUM_i) - G \cdot HUM_i) 
\leq b_i \delta_t
\]

for the HV of refuse coming from the separators (the fraction without organic material)

\[
\sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{n=1}^{N} \sum_{i=0}^{6} (1 - \alpha_i) \varphi'_{P_d,i} \varphi_{P_d,i} (1 - k_i)(1 - \eta_{S_p,i}) \varphi_{S_p,i} (1 - \eta_{C_q,i}) \varphi_{C_q,i} (HV_i(1 - HUM_i) - G \cdot HUM_i) 
\leq b_i \delta_t
\]

for the scraps coming from RDF-plants.
\[
+ \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{n=1}^{N} \sum_{i=0}^{6} (1 - \alpha_i) r_i \chi_{P_d,i} \varphi_{P_d,S_p} (1 - k_i)(1 - \eta_{S_p,i}) \psi_{S_p,C_q} (1 - \hat{\eta}_{C_q,i}) \\
\times \xi_{T_n} \sigma_{I_n,I_m} \lambda_{C_q,I_n} \\
+ \sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{i=0}^{6} (1 - \alpha_i) r_i \chi_{P_d,i} \varphi_{P_d,S_p} (1 - k_i)\eta_{S_p,i}(1 - \hat{\eta}_{T_n,i}) \gamma_{T_n,I_n} \leq \frac{M_{L_{m,x}}}{T_{L_{m,y}}} n,
\]

where \( M_{L_{m,x}} \) is the (mass) quantity of waste that saturates landfill \( L_{m} \) (tons); \( T_{L_{m,y}} \), minimum allowed time to saturate landfill \( L_{m} \) (years); \( n \), number of days in a year in which the refuse is collected; \( \xi_{I_n} \), mass percentage of the refuse entering the incinerator that is transformed into ashes.

### 4.2.4. Mass balance equations

Mass conservation equations are needed for each branching point at which a flow can be split. Such equations are:

\[
\sum_{m=1}^{M} \varphi_{P_d,L_m} + \sum_{n=1}^{N} \varphi_{P_d,I_n} + \sum_{p=1}^{P} \varphi_{P_d,S_p} = 1 \quad d = 1, \ldots, D,
\]

\[
\sum_{m=1}^{M} \psi_{S_p,L_m} + \sum_{n=1}^{N} \psi_{S_p,I_n} + \sum_{q=1}^{Q} \psi_{S_p,C_q} = \delta_{S_p} \quad p = 1, \ldots, P,
\]

\[
\sum_{m=1}^{M} \lambda_{C_q,L_m} + \sum_{n=1}^{N} \lambda_{C_q,I_n} = \delta_{C_q} \quad q = 1, \ldots, Q,
\]

\[
\sum_{m=1}^{M} \sigma_{I_n,L_m} = \delta_{I_n} \quad n = 1, \ldots, N.
\]

### 4.2.5. Constraints imposing the presence of plants

A constraint must be introduced for every plant whose presence/absence is a matter of the decision problem. This constraint must impose that if the flow entering such a plant is greater than zero, then the plant must actually exist. In our model, such constraints must be introduced for every separator, any RDF-plant, any incinerator, and any landfill. Note that it is not necessary to impose such constraints for any organic material plant, since it is assumed that there is a one-to-one correspondence of such plants with separators. In summary, we have:

\[
\sum_{d=1}^{D} \varphi_{P_d,S_p} - \delta_{S_p} A \leq 0 \quad p = 1, \ldots, P,
\]

\[
\sum_{p=1}^{P} \psi_{S_p,C_q} - \delta_{C_q} A \leq 0 \quad d = 1, \ldots, D,
\]
\[
\sum_{d=1}^{D} \varphi_{p_d} I_n + \sum_{p=1}^{P} \psi_{S_p} I_n + \sum_{q=1}^{Q} \lambda_{C_q} I_n - \delta I_n A \leq 0 \quad n = 1, \ldots, N, \\
\sum_{d=1}^{D} \varphi_{p_d} I_n + \sum_{p=1}^{P} \psi_{S_p} I_n + \sum_{q=1}^{Q} \lambda_{C_q} I_n + \sum_{n=1}^{N} \sigma_{I_n} I_n - \delta I_n A \leq 0 \quad m = 1, \ldots, M,
\]

where \( A \) is a very great number with respect to values that can be assumed by every decision variable.

4.2.6. Bounds on the flows entering some kinds of plants

In some cases, technical reasons impose upper and lower bounds on the flow entering separators and RDF-plants must be taken into account, that is:

\[
M_{a,S_p} \delta_{S_p} \leq \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i) r_i \lambda_{p_d,i} \varphi_{p_d,S_p} \leq M_{b,S_p} \delta_{S_p} \quad p = 1, \ldots, P,
\]

\[
M_{a,C_q} \delta_{C_q} \leq \sum_{d=1}^{D} \sum_{q=1}^{Q} \sum_{i=0}^{6} (1 - \alpha_i) r_i \lambda_{p_d,i} \varphi_{p_d,S_p} (1 - \eta_{S_p,i}) \psi_{S_p,C_q} \leq M_{b,C_q} \delta_{C_q}
\]

\[q = 1, \ldots, Q.\]

4.3. The objective function

The objective function to be minimized takes into account recycling, transportation, and plant costs, along with possible economic benefits. More specifically:

- recycling costs take into account separate collection costs; thus, such costs depend on the way separate collection is actually carried out (ecological islands, directly at home, different types of containers positioned on the road), and on possible incomes (coming from government contributions) that are given to favor recycling and are in general different for the various materials; note that, contrarily to intuition, recycling, although it provides materials to be sold, actually yields a net cost, from the point of view of the company that has the responsibility of waste management and disposal in a metropolitan area; that is due to the fact that in general such a company pays private companies which actually perform separate collection (such costs usually overcome the economic benefits provided by government);
- transportation costs depend on the number of vehicles involved, on personnel costs, and on the distances among the various plants; note that collection costs relevant to waste gathering at the various collection sites are not considered in our model, as they do not depend on the decision variables of the problem;
- plant costs take into account installation and maintenance costs;
- economic benefits include money gained from selling electric energy and RDF.
4.3.1. Recycling costs

Recycling can take place through different techniques $j = 1, \ldots, 4$ (1 is referred to collection directly at home, 2 through ecological island, 3 through special holders and 4 using small holders). Fixed parameters $\omega_{ij}$ represent the fraction of material $i$ collected through method $j$ (with respect to the total quantity of material $i$ sent to recycling). Let $C_{ij}^c$ be the unit collection cost for material $i$ and technique $j$, and $C_i$ the unit benefit coming from government contributions. Then, the annual recycling cost can be expressed as:

$$C^c = n \sum_{i=1}^{6} \left( \sum_{j=1}^{4} C_{ij}^c \alpha_i \omega_{ij} \right) - C_i \alpha_i.$$

(18)

In this formulation, the plant costs for recycling and benefits coming from the sale of recycled material are not taken into account since private companies manage these aspects. In general, as mentioned above, $C^c$ turns out to assume a positive value. Note that in Eq. (18) coefficients $\omega_{ij}$ are assumed to be fixed and known.

4.3.2. Transportation costs

Transportation costs include:

– the number of necessary employees which depends on the number of trips necessary to transport waste and on the number of trips that a single driver can do during his work-day);
– the vehicle costs;
– the possible tolls to be paid.

The overall annual transportation cost $C^t$ results from the sum of all considered transportation costs:

a) from collecting sites to landfills; the annual waste volume ($n$ is the number of working day in a year) sent from collecting site $P_d$ to landfill $L_m$ (having density $\rho_{P_d,L_m}$) is given by:

$$\hat{Q}_{P_d,L_m} = n \sum_{i=0}^{6} (1 - \alpha_i)r_i\lambda_{P_d,L_m} \theta_{P_d,L_m}/\rho_{P_d,L_m}.$$

(19a)

moreover let $\hat{Q}_{P_d,L_m} = \hat{Q}_{P_d,L_m} \rho_{P_d,L_m}$ be the annual mass flow sent from collecting site $P_d$ to landfill $L_m$; then the overall transportation cost from collecting sites to landfills is:

$$\sum_{d=1}^{D} \sum_{m=1}^{M} \frac{\hat{Q}_{P_d,L_m} C_{P_d,L_m}}{V_{P_d,L_m}},$$

where $C_{P_d,L_m}$ is the cost of a trip and $V_{P_d,L_m}$ is the capacity of a single vehicle;

b) from collecting sites to incinerators; the annual waste volume transported from collecting site $P_d$ to incinerator $I_n$ (having density $\rho_{P_d,I_n}$) is given by
\[
\hat{Q}_{p_i,t_n} = \sum_{i=0}^{6} (1 - \alpha_i) r_{i} \phi_{p_i,t_n} \phi_{p_i,t_n} / \rho_{p_i,t_n};
\]  
(19b)

and the transportation cost can be determined in the same way as above;

c) from separators to RDF plants; the annual waste volume transported from separator \(S_p\) to RDF plant \(C_q\) (having density \(\rho_{S_p,C_q}\)) is given by:

\[
\hat{Q}_{S_p,C_q} = n \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i) r_{i} \phi_{p_d,S_p} (1 - k_i) (1 - \eta_{S_p,i}) \psi_{S_p,C_q} / \rho_{S_p,C_q};
\]  
(19c)

d) from separators to organic material treatment plants; the annual waste volume transported from separator \(S_p\) to the plant for organic material (having density \(\rho_{S_p}\)) is given by:

\[
\hat{Q}_{S_p} = n \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i) r_{i} \phi_{p_d,S_p} (1 - k_i) \eta_{S_p,i} / \rho_{S_p};
\]  
(19d)

e) from separators to landfills; the annual waste volume transported from separator \(S_p\) to landfill \(L_m\) (having density \(\rho_{S_p,L_m}\)) is given by:

\[
\hat{Q}_{S_p,L_m} = n \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i) r_{i} \phi_{p_d,S_p} (1 - k_i) (1 - \eta_{S_p,i}) \psi_{S_p,L_m} / \rho_{S_p,L_m};
\]  
(19e)

f) from separators to incinerators; the annual waste volume transported from separator \(S_p\) to incinerator \(I_n\) (having density \(\rho_{S_p,I_n}\)) is given by:

\[
\hat{Q}_{S_p,I_n} = n \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i) r_{i} \phi_{p_d,S_p} (1 - k_i) (1 - \eta_{S_p,i}) \psi_{S_p,I_n} / \rho_{S_p,I_n};
\]  
(19f)

g) from RDF plants to landfills; the annual waste volume transported from RDF plant \(C_q\) to landfill \(L_m\) (having density \(\rho_{C_q,L_m}\)) is given by:

\[
\hat{Q}_{C_q,L_m} = n \sum_{p=1}^{P} \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i) r_{i} \phi_{p_d,S_p} (1 - k_i) (1 - \eta_{C_q,i}) \lambda_{C_q,L_m} / \rho_{C_q,L_m};
\]  
(19g)

h) from RDF plants to incinerators; the annual waste volume transported from RDF plant \(C_q\) to incinerator \(I_n\) (having density \(\rho_{C_q,I_n}\)) is given by:
\[
\hat{Q}_{C_q,l_n} = n \sum_{p=1}^{P} \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i)r_i \mathcal{X}_{P_d,i} \varphi_{P_d} s_i (1 - k_i)
\times (1 - \eta_{S_p,i}) \psi_{S_p,C_q} (1 - \hat{\eta}_{C_q}) \lambda_{C_q,l_n}/\rho_{C_q,l_n}; \quad (19h)
\]

i) from incinerators to landfills; the annual waste volume transported from incinerator \(I_n\) to landfill \(L_m\) \((\hat{\xi}_{L_m}\) is the volume fraction of waste entering the incinerator that comes out as ashes) is given by:

\[
\hat{Q}_{I_n,L_m} = n \hat{\xi}_{I_n} \left( \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i)r_i \mathcal{X}_{P_d,i} \varphi_{P_d} s_i (1 - k_i)(1 - \eta_{S_p,i}) \psi_{S_p,l_n}/\rho_{S_p,l_n} \right) + \sum_{i=0}^{6} (1 - \alpha_i)r_i \mathcal{X}_{P_d,i} \varphi_{P_d} s_i (1 - k_i)
\times (1 - \eta_{S_p,i}) \psi_{S_p,C_q} (1 - \hat{\eta}_{C_q}) \lambda_{C_q,l_n}/\rho_{C_q,l_n} \sigma_{l_n,L_m}; \quad (19i)
\]

j) from collecting sites to separators; the annual waste volume transported from collecting site \(P_{d}\) to separator \(S_{p}\) (having density \(\rho_{P_{d},S_{p}}\)) is given by:

\[
\hat{Q}_{P_{d},S_{p}} = n \sum_{i=0}^{6} (1 - \alpha_i)r_i \mathcal{X}_{P_d,i} \varphi_{P_d} s_i (1 - k_i)/\rho_{P_{d},S_{p}}; \quad (19j)
\]

k) from organic material plants to landfills; the annual waste volume transported from organic material plant \(T_{p}\) to landfill \(L_m\) (having density \(\rho_{T_{p},L_m}\)) is given by:

\[
\hat{Q}_{T_{p},L_m} = n \sum_{d=1}^{D} \sum_{i=0}^{6} (1 - \alpha_i)r_i \mathcal{X}_{P_d,i} \varphi_{P_d} s_i (1 - k_i) \eta_{S_p,T_{p}} \mathcal{Y}_{T_p,L_m}/\rho_{T_{p},L_m}. \quad (19k)
\]

Thus the expression of the overall transportation cost is the following:

\[
C^t = \sum_{d=1}^{D} \sum_{m=1}^{M} \frac{\hat{Q}_{P_{d},L_m}}{V_{P_{d},L_m}} C_{P_{d},L_m} + \sum_{d=1}^{D} \sum_{n=1}^{N} \frac{\hat{Q}_{I_n,l_n}}{V_{P_{d},L_n}} C_{P_{d},l_n} + \sum_{p=1}^{P} \sum_{q=1}^{Q} \frac{\hat{Q}_{S_p,C_q}}{V_{S_p,C_q}} C_{S_p,C_q} + \sum_{p=1}^{P} \frac{\hat{Q}_{S_p,C_q}}{V_{S_p}} C_{S_p}^t
\]
\[ + \sum_{p=1}^{P} \sum_{m=1}^{M} \frac{\dot{Q}_{S_p,I_m}}{V_{S_p,I_m}} C_{S_p,I_m} + \sum_{p=1}^{P} \sum_{n=1}^{N} \frac{\dot{Q}_{S_p,I_n}}{V_{S_p,I_n}} C_{S_p,I_n} \]
\[ + \sum_{q=1}^{Q} \sum_{m=1}^{M} \frac{\dot{Q}_{C_q,I_m}}{V_{C_q,I_m}} C_{C_q,I_m} + \sum_{n=1}^{N} \sum_{m=1}^{M} \frac{\dot{Q}_{I_m,I_n}}{V_{I_m,I_n}} C_{I_m,I_n} \]
\[ + \sum_{d=1}^{D} \sum_{p=1}^{P} \frac{\dot{Q}_{P_d,S_p}}{V_{P_d,S_p}} C_{P_d,S_p} + \sum_{q=1}^{Q} \sum_{n=1}^{N} \frac{\dot{Q}_{C_q,I_n}}{V_{C_q,I_n}} C_{C_q,I_n} + \sum_{p=1}^{P} \sum_{m=1}^{M} \frac{\dot{Q}_{T_p,I_m}}{V_{T_p,I_m}} C_{T_p,I_m}, \quad (19n) \]

where symbols not yet defined have an obvious meaning.

### 4.3.3. Maintenance costs

The cost function is characterized by fixed and variable costs. In particular, fixed costs are multiplied by the integer decision variable relative to the specific plant. The following expression represents maintenance costs for all plants in the system.

\[
C^g = \sum_{p=1}^{P} \sum_{d=1}^{D} \bar{Q}_{P_d,S_p} C_{S_p} + \sum_{p=1}^{P} C_{F,S_p} \delta_{S_p} + \sum_{p=1}^{P} \sum_{q=1}^{Q} \bar{Q}_{C_q,S_p} C_{C_q} + \sum_{q=1}^{Q} C_{F,C_q} \delta_{C_q} \\
+ \sum_{p=1}^{P} \bar{Q}_{S_p} C_{I_p} + \sum_{p=1}^{P} C_{F,I_p} \delta_{I_p} + \sum_{n=1}^{N} \left( \sum_{d=1}^{D} \bar{Q}_{P_d,I_m} + \sum_{p=1}^{P} \bar{Q}_{S_p,I_n} + \sum_{q=1}^{Q} \bar{Q}_{C_q,I_n} \right) C_{I_m} \\
+ \sum_{n=1}^{N} C_{F,I_n} \delta_{I_n} + \sum_{m=1}^{M} \left( \sum_{d=1}^{D} \bar{Q}_{P_d,I_m} + \sum_{p=1}^{P} \bar{Q}_{S_p,I_m} + \sum_{q=1}^{Q} \bar{Q}_{C_q,I_m} + \sum_{n=1}^{N} \bar{Q}_{I_n,I_m} \right) C_{I_m} \\
+ \sum_{m=1}^{M} C_{F,I_m} \delta_{I_m}, \quad (20) \]

where \( C_{F,z} \) represents the annual fixed cost for the plant \( z \), \( C_z \) is the unit treatment cost in plant \( z \), and \( \bar{Q}_{z,w} \) is the annual flow (in mass) sent from plant \( z \) to \( w \).

### 4.3.4. The overall objective function

Finally, the cost function takes into account possible benefits as a result either of electric energy production or of RDF selling. Specifically, annual energy recovery benefit \( C_{I_n}^{\text{b}} \) has to be taken into account only whenever the incinerator \( I_n \) is actually present. Note that \( C_{I_n}^{\text{b}} \) is a constant because it is assumed that the size and the kind of incinerator \( I_n \) are a priori fixed. Every kg of produced RDF can be sold at the price \( C_C \), which is known. Then the overall possible benefits can be written as:

\[
B = \sum_{q=1}^{Q} n C_{C,RDF} C_q + \sum_{n=1}^{N} C_{I_n}^{\text{b}} \delta_{I_n}, \quad (21) \]

where \( RDF_{C_q} \) is defined as in Eq. (2a).
Then, by using the previously defined terms, the overall cost function may be written as:

\[ C = C^r + C^t + C^g - B. \]  

(22)

5. The considered case study

The model proposed has been applied to the case study concerning the municipality of Genova where refuse disposal is a very critical problem. With a daily waste production of 1355 tons of which only 18% is presently recycled, the current solution is disposal in a unique landfill, whose residual capacity is rapidly decreasing. Moreover, Italian legislation requires that the percentage of recycled waste must reach 35% within 2003, and strongly discourages disposal of untreated refuse in landfills. Table 1 reports unit costs for separate collection and government contributions for all the considered materials.

Presently, the possible alternative solutions that can be designed and evaluated are based on a set of already existing plants and on a set of ‘potential plants’, namely:

- two collecting sites \((D = 2)\);
- two possible separators \((P = 2)\); they are able to recover only metals \((k_6 = 0.9; k_i = 0, i = 0, \ldots, 5)\);
- one possible incinerator \((N = 1)\);
- one landfill \((M = 1)\);
- one possible RDF plant \((Q = 1)\);
- one possible plant for organic material.

For each of the potential plants, site, size, and technologies have already been fixed, so that all parameters in the cost function can be assumed as known.

Note that, on the whole, there are five potential plants and hence five integer decision variables. Moreover, in the case study, there are 14 continuous decision

### Table 1
Unit costs for separate collection and government contributions for recycling for the considered case study

<table>
<thead>
<tr>
<th>Unit benefits ((€/t)) (C_i)</th>
<th>Paper</th>
<th>Plastic</th>
<th>Glass</th>
<th>Organic</th>
<th>Wood</th>
<th>Metals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directly at home (C_i)</td>
<td>15</td>
<td>70</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>Ecological island (C_i)</td>
<td>115</td>
<td>155</td>
<td>–</td>
<td>125</td>
<td>60</td>
<td>250</td>
</tr>
<tr>
<td>Special holders (C_i)</td>
<td>30</td>
<td>80</td>
<td>10</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Small holders (C_i)</td>
<td>16</td>
<td>48</td>
<td>62</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>152</td>
<td>142</td>
</tr>
</tbody>
</table>
variables: six $x_i$, three $\varphi_{z,w}$, three $\psi_{z,w}$, two $\lambda_{z,w}$. Note that it is not necessary to introduce neither variables $\sigma_z$ nor variables $\gamma_z$ as there is only one landfill.

The optimization problem has been solved on a Pentium III 600 MHz computer by use of the optimization software Lingo 6.0 (LINDO Systems, Inc., http://www.lindo.com) requiring less than one minute of computation.

As the main problem troubling the planners seems to be the rapid landfill saturation, the optimization problem has been solved in four different versions:

- Case 1a: constraint (7) about the quantity of material that can be sent to the landfill is not taken into account;
- Case 1b: constraint (7) is taken into account, for several different specification of parameter $T_R$ (minimum allowed filling time for the sanitary landfill);
- Case 2a: a constraint is added to those introduced in the previous section, forbidding the disposal of untreated refuse in the landfill, i.e. setting $\sum_{d=1}^2 \varphi_{P_d,L_i} = 0$; instead, constraint (7) is not taken into account;
- Case 2b: as Case 2a, but with constraint (7) taken into account.

In Table 2, the results obtained for Cases 1a and 2a are shown and compared with the present management policy. Obviously, the optimal annual cost obtained in Case 1a are lower than the annual costs obtained in Case 2a. It turns out that in Case 1a the incinerator is not present in the optimal solution, contrarily to Case 2a. It is interesting to note that, following the optimal solution determined for Case 1a, the landfill saturates in 3 years, whereas in the optimal solution determined for Case 2a, the landfill saturates in 10 years.

The results relevant to Case 2b are reported in Table 3, for different values of parameter $T_R$ (8, 20 and 40 years). For $T_R = 8$ or 20 years, all plants mentioned at the beginning of this section are necessary, whereas for a higher minimum filling time of the landfill, $T_R = 40$ years, the incinerator is not present in the optimal solution and costs considerably increase. This is due to the fact that the incinerator produces ashes that must be deposited in landfill and if the objective is to reduce the flow to the landfill, then it is necessary to increase recycling and material recovery. Of course, the cost of the optimal solution increases as $T_R$ grows, as recycling is very expensive.

Finally, Fig. 3 shows a sensitivity analysis with respect to the choice of parameter $T_R$. The optimal values of the two decision variables $\varphi_S = \sum_{d=1}^2 \sum_{p=1}^2 \varphi_{P_d,S_p}$ and $\varphi_I = \sum_{d=1}^2 \varphi_{P_d,L_i}$, represent the fraction of untreated waste that is sent to the separators and the fraction sent to the incinerator, respectively. The optimal costs are plotted in Fig. 4, again as function of parameter $T_R$.

Then, the following conclusions can be drawn:

- despite its low optimal costs, Case 1a is not sustainable from the environmental point of view;
- two separators have to be built, as well as one plant for organic material treatment and one RDF plant;
Table 2
Results obtained for Cases 1a and 2a

<table>
<thead>
<tr>
<th></th>
<th>Present values</th>
<th>Optimal values Case 1a</th>
<th>Optimal values Case 2a</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1$ (Separator 2)</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_2$ (RDF-plant)</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_3$ (organic material-plant)</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_4$ (incinerator)</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_5$ (landfill)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_6$ (Separator 1)</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$x_1$ (paper)</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$x_2$ (plastic)</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$x_3$ (glass)</td>
<td>60</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>$x_4$ (organic)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_5$ (wood)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$x_6$ (metals)</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\varphi_S$</td>
<td>0</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>$\varphi_I$</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>$\psi_C$</td>
<td>1</td>
<td>0.7</td>
<td>–</td>
</tr>
<tr>
<td>$\psi_L$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_I$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_L$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_I$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RDF (tons/day)</td>
<td>0</td>
<td>175</td>
<td>200</td>
</tr>
<tr>
<td>Recovered organic material (tons/day)</td>
<td>0</td>
<td>110</td>
<td>380</td>
</tr>
<tr>
<td>Metals (tons/day)</td>
<td>0</td>
<td>18</td>
<td>35</td>
</tr>
<tr>
<td>Total costs (k€/year)</td>
<td>18 500</td>
<td>22 000</td>
<td>30 500</td>
</tr>
</tbody>
</table>

**Untreated waste destination**

Fig. 3. Untreated waste destination for the considered four cases.
Table 3
Results obtained for Case 2b

<table>
<thead>
<tr>
<th>Case 2b</th>
<th>$T_R$ (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td>$\delta_1$ (Separator 2)</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_2$ (RDF-plant)</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_3$ (organic material-plant)</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_4$ (incinerator)</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_5$ (landfill)</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_6$ (Separator 1)</td>
<td>1</td>
</tr>
<tr>
<td>$\pi_1$ (paper)</td>
<td>15</td>
</tr>
<tr>
<td>$\pi_2$ (plastic)</td>
<td>8</td>
</tr>
<tr>
<td>$\pi_3$ (glass)</td>
<td>80</td>
</tr>
<tr>
<td>$\pi_4$ (organic)</td>
<td>0</td>
</tr>
<tr>
<td>$\pi_5$ (wood)</td>
<td>10</td>
</tr>
<tr>
<td>$\pi_6$ (metals)</td>
<td>2</td>
</tr>
<tr>
<td>$\phi_X$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\phi_C$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\psi_L$</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_I$</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_L$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_I$</td>
<td>0</td>
</tr>
<tr>
<td>RDF (tons/day)</td>
<td>200</td>
</tr>
<tr>
<td>Recovered organic material (tons/day)</td>
<td>380</td>
</tr>
<tr>
<td>Metals (tons/day)</td>
<td>35</td>
</tr>
<tr>
<td>Total costs (k€/year)</td>
<td>30 500</td>
</tr>
</tbody>
</table>

Fig. 4. Optimal annual costs in Cases 1a and 2a.
in connection with Case 2b, two possible realistic and sustainable alternatives can be proposed to build one incinerator, spending 32,000 € per year, and saturating the landfill in 20 years; not to build any incinerator, spending 38,000 € per year, with a very strong recycling effort, and saturating the landfill in 40 years.

6. Conclusions and future directions

The proposed DSS allows the planning of the treatment plants that must be used in an optimal MSW management system and defines how to size recycling and waste disposal in an integrated approach. The DSS is based on the formalization of a constrained non-linear optimization problem, where some decision variables are binary and other ones are continuous. The objective function includes all possible economical costs, whereas constraints arise from technical, normative, and environmental issues.

On the whole, the proposed approach allows taking into account a multiplicity of aspects and issues which play a crucial role in planning MSW management systems. A careful attention has been paid to provide a proper characterization of the system, as regards waste composition, heating value, material recovery, and possible treatments.

Possible future developments should tend to the objective of extending the proposed model in order to allow it to include an analytical representation of environmental impact (atmospheric pollution, impact of the city traffic, etc.). In this direction, some suggestions can come from some recent works. For instance Tsiliyannis (1999) has discussed the main environmental problems related to MSW management, and in particular pollutant releases. Chang and Wang (1997a), have proposed a fuzzy goal programming approach for optimal planning of MSW systems, in which they consider four objectives: economical costs, noise control, air pollution control, and traffic congestion limitations. Another possible approach is based on life cycle assessment (Finnveden, 1996; Barton et al., 2000). Specifically, Finnveden (1996) has discussed some methodological issues that arise in connection with such an approach. Moreover, the waste reduction problem could be studied at a more general level, including all aspects reporting production processes (Young et al., 1996).

Another possible development of this work may regard the refinement of the analysis of recycling, transportation and maintenance costs; for example, the simplifying assumption that such costs are linear in the flows could be realistically removed.

In this connection, a deeper correlation with the territorial system should be taken into account. For instance, separate collection costs may depend not only on the different kinds of materials, but also on the different areas of the considered municipality. In such a case separate collection should be carefully sized for the different areas.

Further, possible extensions could regard the introduction of decision variables concerning the size and the technical characteristics of the plants which are included
in the configuration of the MSW management system determined by the DSS. In fact, in the model proposed in this paper, such plants (even those whose actual presence is a matter of the decision problem) are assumed to have already been decided. This may be not realistic in all cases in which the determination of optimal refuse flows directly influences the characteristics and sizes of such plants.

Finally, another possibility would be that of formalizing the decision problem in a dynamical setting, i.e. considering time-varying (over years) input refuse flows, and a time-varying configuration of the MSW management system. In such a way, it would be possible to determine optimal sequence of interventions (capacity expansions, building of new plants, etc.), over a given time horizon, capable of optimally driving the MSW management system from the present configuration to a final one.

Acknowledgements

The authors would like to thank Amiu (Azienda Multiservizi di Igiene Urbana), the company managing MSW in Genova municipality, Italy, for the precious contributions to the development of this work. A Lingo batch file implementing the mathematical model described in the result section of this work is available by sending a request by e-mail to the corresponding author.

Appendix A: List of Notations

**Decision variables**

- $a_0$: 0; inert matter and scraps are assessed as not recyclable
- $a_1$: Percentage of recycled plastic
- $a_2$: Percentage of recycled paper
- $a_3$: Percentage of recycled glass
- $a_4$: Percentage of recycled organic material
- $a_5$: Percentage of recycled wood
- $a_6$: Percentage of recycled metals
- $\varphi_{P_d\cdot L_m}$: fraction of untreated waste coming from collecting site $P_d$ $(d = 1, \ldots, D)$ and sent to landfill $L_m$ $(m = 1, \ldots, M)$
- $\varphi_{P_d\cdot S_p}$: fraction of untreated waste coming from collecting site $P_d$ $(d = 1, \ldots, D)$ and sent to separator $S_p$ $(p = 1, \ldots, P)$
- $\varphi_{P_d\cdot I_n}$: fraction of untreated waste coming from collecting site $P_d$ $(d = 1, \ldots, D)$ and sent to incinerator $I_n$ $(n = 1, \ldots, N)$
- $\psi_{S_p\cdot C_q}$: fraction of material coming from separator $S_p$ $(p = 1, \ldots, P)$ and sent to RDF-plant $C_q$ $(q = 1, \ldots, Q)$
- $\psi_{S_p\cdot I_n}$: fraction of material coming from separator $S_p$ $(p = 1, \ldots, P)$ and sent to incinerator $I_n$ $(n = 1, \ldots, N)$
- $\psi_{S_p\cdot L_m}$: fraction of material coming from separator $S_p$ $(p = 1, \ldots, P)$ and sent to landfill $L_m$ $(m = 1, \ldots, M)$
\( \lambda_{C_q, L_m} \)  
> fraction of scraps coming from RDF-plant  
> \( C_q \) \((q = 1, \ldots, Q)\) and sent to landfill  
> \( L_m \) \((m = 1, \ldots, M)\)

\( \lambda_{C_q, I_n} \)  
> fraction of scraps coming from RDF-plant  
> \( C_q \) \((q = 1, \ldots, Q)\) and sent to incinerator  
> \( I_n \) \((n = 1, \ldots, N)\)

\( \sigma_{I_n, L_m} \)  
> fraction of ashes coming from incinerator  
> \( I_n \) \((n = 1, \ldots, N)\) and sent to landfill  
> \( L_m \) \((m = 1, \ldots, M)\)

\( \gamma_{T_p, L_m} \)  
> fraction of not recovered waste coming from organic  
> material plant  \( T_p \) \((p = 1, \ldots, P)\) and sent to landfill  
> \( L_m \) \((m = 1, \ldots, M)\)

\( \delta_{S_p} \)  
> binary decision variable related to separator  
> \( S_p \) \((p = 1, \ldots, P)\)

\( \delta_{C_q} \)  
> binary decision variable related to RDF plant  
> \( C_q \) \((q = 1, \ldots, Q)\)

\( \delta_{I_n} \)  
> binary decision variable related to incinerator  
> \( I_n \) \((n = 1, \ldots, N)\)

\( \delta_{L_m} \)  
> binary decision variable related to landfill  
> \( L_m \) \((m = 1, \ldots, M)\)

**Other notations**

- \( P \)  
> number of separators
- \( D \)  
> number of collecting sites
- \( Q \)  
> number of RDF-plants
- \( N \)  
> number of incinerators
- \( M \)  
> number of landfills
- \( \text{RDF}_{C_q} \)  
> daily quantity of produced RDF in  
> \( C_q \)
- \( \hat{Q}_{P_d, L_m} \)  
> annual refuse volume transported from collecting site  
> \( P_d \) to landfill  
> \( L_m \)
- \( \hat{Q}_{P_d, I_n} \)  
> annual refuse volume transported from collecting site  
> \( P_d \) to incinerator  
> \( I_n \)
- \( \hat{Q}_{S_p, C_q} \)  
> annual refuse volume transported from separator  
> \( S_p \) to RDF plant  
> \( C_q \)
- \( \hat{Q}_{S_p} \)  
> annual refuse volume transported from separator  
> \( S_p \) to the plant for organic material
- \( \hat{Q}_{S_p, L_m} \)  
> annual refuse volume transported from separator  
> \( S_p \) to landfill  
> \( L_m \)
- \( \hat{Q}_{S_p, I_n} \)  
> annual refuse volume transported from separator  
> \( S_p \) to incinerator  
> \( I_n \)
- \( \hat{Q}_{C_q, L_m} \)  
> annual refuse volume transported from RDF plant  
> \( C_q \) to landfill  
> \( L_m \)
- \( \hat{Q}_{I_n, L_m} \)  
> annual refuse volume transported from incinerator  
> \( I_n \) to landfill  
> \( L_m \)
- \( \hat{Q}_{P_d, S_p} \)  
> annual refuse volume transported from collecting site  
> \( P_d \) to separator  
> \( S_p \)
- \( \hat{Q}_{C_q, L_m} \)  
> annual refuse volume transported from RDF plant  
> \( C_q \) to landfill  
> \( L_m \)
\( \hat{Q}_{T_p,L_m} \) annual refuse volume transported from organic material plant \( T_p \) to landfill \( L_m \)

\( \hat{Q}_{P_d,S_p} \) annual mass flow of material coming from collecting site \( P_d \) treated in the separator \( S_p \)

\( \hat{Q}_{C_q,S_p} \) annual mass flow of refuse coming from separator \( S_p \) and treated in the RDF plant \( C_q \)

\( \hat{Q}_{S_p} \) annual mass flow of material coming from the separator and treated in the plant for organic material

\( \hat{Q}_{P_d,I_n}, \hat{Q}_{S_p,I_n} \) and \( \hat{Q}_{C_q,I_n} \) annual refuse flows sent to incinerator plant \( I_n \) coming from, respectively, collection site \( P_d \), separator \( S_p \) and RDF plant \( C_q \)

\( \hat{Q}_{P_d,L_m}, \hat{Q}_{S_p,L_m} \) and \( \hat{Q}_{C_q,L_m} \) annual refuse flows sent to landfill \( L_m \) coming from, respectively, collection site \( P_d \), separator \( S_p \) and RDF plant \( C_q \)

\( \gamma_{P_d,I} \) percentage of material \( i \) sent to collection site \( P_d \)

\( C_{ij} \) unit costs (€/t) for the \( j \)th method of collection and for the \( i \)th material

\( \omega_{i,j} \) fraction of material \( i \) collected through method \( j \)

\( C_I \) contributions for recycling the \( i \)th material (€/t)

\( C_{P_d,I_m} \) cost of a trip from collection site \( P_d \) to landfill \( L_m \)

\( V_{P_d,I_m} \) capacity of the vehicle used for the trip from collection site \( P_d \) to landfill \( L_m \)

\( C_{P_d,I_n} \) cost of a trip from collection site \( P_d \) to incinerator \( I_n \)

\( V_{P_d,I_n} \) capacity of the vehicle used for the trip from collection site \( P_d \) to incinerator \( I_n \)

\( C_{S_p} \) cost of a trip from collection separator \( S_p \) to the associated organic material plant

\( V_{S_p} \) capacity of the vehicle used for the trip from separator \( S_p \) to the associated organic material plant

\( C_{S_p,I_m} \) cost of a trip from separator \( S_p \) to landfill \( L_m \)

\( V_{S_p,I_m} \) capacity of the vehicle used for the trip from separator \( S_p \) to landfill \( L_m \)

\( C_{S_p,I_n} \) cost of a trip from separator \( S_p \) to incinerator \( I_n \)

\( V_{S_p,I_n} \) capacity of the vehicle used for the trip from separator \( S_p \) to incinerator \( I_n \)

\( C_{C_q,I_m} \) cost of a trip from RDF-plant \( C_q \) to landfill \( L_m \)

\( V_{C_q,I_m} \) capacity of the vehicle used for the trip from RDF-plant \( C_q \) to landfill \( L_m \)

\( C_{I_n,L_m} \) cost of a trip from incinerator \( I_n \) to landfill \( L_m \)

\( V_{I_n,L_m} \) capacity of the vehicle used for the trip from incinerator \( I_n \) to landfill \( L_m \)

\( C_{P_d,S_p} \) cost of a trip from collection site \( P_d \) to separator \( S_p \)

\( V_{P_d,S_p} \) capacity of the vehicle used for the trip from collection site \( P_d \) to separator \( S_p \)

\( C_{C_q,I_n} \) cost of a trip from RDF-plant \( C_q \) to incinerator \( I_n \)
capacity of the vehicle used for the trip from RDF-plant $C_q$ to incinerator $I_n$

cost of a trip from organic material plant $T_p$ to landfill $L_m$

capacity of the vehicle used for the trip from organic material plant $T_p$ to landfill $L_m$

unit cost for separator $S_p$ comprehensive of installation and maintenance costs

unit cost for RDF plant $C_q$ comprehensive of installation and maintenance costs

unit cost for the organic material plant comprehensive of installation and maintenance costs

unit cost for incinerator $I_n$ comprehensive of installation and maintenance costs

unit cost for landfill $L_m$ comprehensive of installation and maintenance costs

number of working days during a year

fixed costs comprehensive of maintenance and installation costs for separator $S_p$

fixed costs comprehensive of maintenance and installation costs for RDF plant $C_q$

fixed costs comprehensive of maintenance and installation costs for organic plant $T_p$

fixed costs comprehensive of maintenance and installation costs for incinerator $I_n$

fixed costs comprehensive of maintenance and installation costs for landfill $L_m$

produced Refuse Derived Fuel

percentage of humidity contained in the $i$th material

heating value for the $i$th material, considering dry the material (kcal/kg)

heat necessary to evaporate 1 kg of water (606.5 kcal/kg)

minimum and maximum calorific value accepted by the specific incinerator $I_n$ (kcal/kg)

unit price for RDF’s sell

benefit for electric energy’s sell for every incinerator plant $I_n$

separation efficiency of separator $S_p$

separation efficiency of RDF plant $C_q$

separation efficiency of organic material

very great number

mass percentage of the refuse entering the incinerator that is transformed into ashes

volume percentage of the refuse entering the incinerator that is transformed into ashes
\( K_I \) fraction of material \( i \) that is separated
\( R \) total daily waste production
\( R_I \) daily refuse production for every component
\( \rho \) general symbol to indicate material density
\( M\text{In},a \) lower bound for the size of the incinerators
\( M\text{In},b \) upper bound for the size of the incinerators
\( MLm,r \) mass quantity of waste that saturates landfill \( L_m \)
\( TLm,r \) minimum allowed time to saturate landfill \( L_m \)
\( M\text{a},Sp \) lower bound for the size of the separators
\( M\text{b},Cq \) upper bound for the size of the RDF plants
\( M\text{a},Sp \) lower bound for the size of the separators
\( M\text{b},Cq \) upper bound for the size of the RDF plants

References